Nº PROYECTO : 3140060             DURACIÓN : 3 años             AÑO ETAPA : 2016
TÍTULO PROYECTO : OPTIMAL CONTROL OF NONCONVEX DIFFERENTIAL INCLUSIONS AND APPLICATIONS
DISCIPLINA PRINCIPAL : ANALISIS
GRUPO DE ESTUDIO : MATEMATICAS
INVESTIGADOR(A) RESPONSABLE : HOANG DINH NGUYEN
DIRECCIÓN :
COMUNA :
CIUDAD : Concepción
REGIÓN : VIII REGION

FONDO NACIONAL DE DESARROLLO CIENTIFICO Y TECNOLOGICO (FONDECYT)
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INFORME FINAL
PROYECTO FONDECYT POSTDOCTORADO

OBJETIVOS

Cumplimiento de los Objetivos planteados en la etapa final, o pendientes de cumplir. Recuerde que en esta sección debe referirse a objetivos desarrollados, NO listar actividades desarrolladas.

<table>
<thead>
<tr>
<th>Nº</th>
<th>OBJETIVOS</th>
<th>CUMPLIMENTO</th>
<th>FUNDAMENTO</th>
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<tr>
<td>1</td>
<td>Optimal control of the sweeping process over controlled sets given by inverse image mappings</td>
<td>TOTAL</td>
<td>This is the joint work with Boris Mordukhovich (Wayne State University, USA). This paper is at the final state of preparation.</td>
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<td>Optimal control problem for a chemostat model with constraints on biomass and nutrient concentrations</td>
<td>TOTAL</td>
<td>This is the joint work with Pedro Gajardo (UTFSM). This paper is under preparation.</td>
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<td>3</td>
<td>Optimal control of the sweeping process over polyhedral controlled sets</td>
<td>TOTAL</td>
<td>This is the joint work with Boris Mordukhovich (Wayne State University, USA), Giovanni Colombo (University of Padova, Italy) and Rene Henrion (Weierstrass Institute for Applied Analysis and Stochastics, Germany). This work is published in Journal of Differential Equations (2016), Vol. 260 (1), 3397-3447.</td>
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Otro(s) aspecto(s) que Ud. considere importante(s) en la evaluación del cumplimiento de objetivos planteados en la propuesta original o en las modificaciones autorizadas por los Consejos.
RESULTS OBTAINED:
For each specific goal, describe or summarize the results obtained. Relate each one to work already published and/or manuscripts submitted. In the Annex section include additional information deemed pertinent and relevant to the evaluation process. The maximum length for this section is 5 pages. (Arial or Verdana, font size 10).

PROJECT FONDECYT No. 3140060: OPTIMAL CONTROL OF NON-CONVEX DIFFERENTIAL INCLUSIONS AND APPLICATIONS

Period: 01/11/2015-31/10/2016

1. Optimal control of the sweeping process over controlled sets given by inverse image mappings
In this the joint work with Boris Mordukhovich (Wayne State University, US). This is a continuity on our last joint work:


In this study, we extend our results obtained in the previous paper from the case of polyhedral sets to sets given by inverse image mappings. This extension allows us to apply our results in the optimal control problem of quasi-static plasticity with linear kinematic hardening with different types of yield functions. In this study, we provide the necessary conditions for minimizers in the type of Pontryagin Maximum Principle. Our study provides one approach to deal with optimal control problems governed by non-Lipschitz dynamics when the certain state constraints are applied. The illustrations of our main results in the area of quasi-static plasticity are included.

This paper is at the final state of preparation.

2. Optimal control problem for a chemostat model with constraints on biomass and nutrient concentrations.
In this the joint work with Pedro Gajardo (UTFSM). This is a continuity on recent works of Pedro Gajardo and others on fed-batch processes:

and


The concept of the chemostat was introduced by Novick, Szilard and Monod in the fifties to describe continuous culture of microorganisms. It describes a culture of microorganisms that can be controlled by the input flow rate. It is widely used in both laboratory and industrial and has a meaning for practical as well as theoretical problems. In our current study, we consider the mathematical dynamical system describing a chemostat (a continuous perfectly mixed bioreactor) with one species growing on one substrate

\[
\begin{aligned}
\dot{x}(t) &= (\mu(s(t)) - u(t))x(t); \quad t \in [0, T] \\
\dot{s}(t) &= -\mu(s(t))x(t) + u(t)(s_{in} - s(t)); \quad t \in [0, T], \\
x(0) &= x_0; \quad s(0) = s_0; \\
0 \leq u(t) \leq u_{\text{max}}; \quad t \in [0, T].
\end{aligned}
\]

(1)

Our objective is to find an optimal feedback control in order to make the whole process be as closed as our target as possible on the fixed time interval \([0, T]\) with restrictions on both biomass and nutrient concentrations:

\[
\begin{aligned}
\int_0^T w_1(x(t) - x_{\text{REF}})^2 + w_2(s(t) - s_{\text{REF}})^2 dt &\rightarrow \min \\
0 < x_m \leq x(t) \leq x_M; \quad 0 < s_m \leq s(t) \leq s_M; \quad t \in [0, T],
\end{aligned}
\]

(2)

This paper is under preparation.

3. Optimal control of the sweeping process over polyhedral controlled sets

This is the joint work with Boris Mordukhovich (Wayne State University, USA), Giovanni Colombo (University of Padova, Italy) and Rene Henrion (Weierstrass Institute for Applied Analysis and Stochastics, Germany). This work is published in Journal of Differential Equations (2016), Vol. 260 (1), 3397-3447.

The paper addresses a new class of optimal control problems governed by the dissipative and discontinuous differential inclusion of the sweeping/Moreau process while using controls to determine the best shape of moving convex polyhedra in order to optimize the given Bolza-type functional, which depends on control and state variables.
as well as their velocities. Besides the highly non-Lipschitzian nature of the unbounded differential inclusion of the controlled sweeping process, the optimal control problems under consideration contain intrinsic state constraints of the inequality and equality types. All of this creates serious challenges for deriving necessary optimality conditions. We develop here the method of discrete approximations and combine it with advanced tools of first-order and second-order variational analysis and generalized differentiation. This approach allows us to establish constructive necessary optimality conditions for local minimizers of the controlled sweeping process expressed entirely in terms of the problem data under fairly unrestrictive assumptions. As a by-product of the developed approach, we prove the strong $W^{1,2}$-convergence of optimal solutions of discrete approximations to a given local minimizer of the continuous-time system and derive necessary optimality conditions for the discrete counterparts. The established necessary optimality conditions for the sweeping process are illustrated by several examples.

4. Existence, stability and optimality for optimal control problems governed by maximal monotone operators

This is the joint work with J. Peypouquet (UTFSM) and L. M. Briceo-Arias (UTFSM). This work is published in Journal of Differential Equations (2016), Vol. 260 (1), 733-757.

We study optimal control problems governed by maximal monotone differential inclusions with mixed control-state constraints in infinite dimensional spaces. We obtain some existence results for this kind of dynamics and construct the discrete approximations that allows us to strongly approximate optimal solutions of the continuous-type optimal control problems by their discrete counterparts. Our approach allows us to apply our results for a wide class of mappings that are applicable in mechanics and material sciences.
ACHIEVEMENTS OF THE PROJECT:
- Research visit(s) to other institution(s).
- Outreach activities related to the projects main topic.
- Any other contribution, not addressed elsewhere, that you consider important.
The maximum length for this section is 1 page. (Arial or Verdana, font size 10).

PROJECT FONDECYT No. 3140060: OPTIMAL CONTROL OF NON-CONVEX DIFFERENTIAL INCLUSIONS AND APPLICATIONS

Period: 01/11/2015-31/10/2016

1. Research visit at University of Science, Hochiminh City, Vietnam (VNUHCM)

Via the invitation of Pr. Dr. Vu Hai Quan, Vice President of VNUHCM-University of Science, during December 27, 2015 to January 17, 2016, I had a research discussion with Prof. Duong Minh Duc and Dr. Nguyen Thi Thu Van of Department of Mathematics and Computer Science and gave one presentation to the Department there (title "Optimal Control Problems with State Constraints: The case of Maximal Monotone Differential Inclusions"). Air ticket of the trip is sponsored by FONDECYT, staying fees (for 20 days) are sponsored by VNUHCM.

Outcomes of the visit: we came up to the memo that in the future, there will be collaboration in advising students of the Master programs of the Department of Mathematics and Computer Science, VNUHCM. In the last ten years, they send over 80 students to the PhD programs in US, France, Italy, Germany, Finland. And via my visit, they want to know more about the possibility for their students to be accepted to universities in Chile.
From: - Prof. Dr. VU Hai Quan  
  Vice president for International Affairs  
  VNUHCM - University of Science  
  Email: vhquan@hcmus.edu.vn

- Dr. HUYNH Quang Vu  
  Vice Dean, Faculty of Mathematics and Computer Science  
  VNUHCM - University of Science  
  Email: hqvu@hcmus.edu.vn

To:  
Prof. NGUYEN Dinh Hoang  
Department of Mathematical Engineering  
University of Concepción, Chile

HoChiMinh City, December 22nd, 2015.

Dear Prof. NGUYEN Dinh Hoang,

Warmest greetings from University of Science, Vietnam National University Ho Chi Minh City.

On behalf of the President of the University of Science, I am delighted to invite you to visit our university in order to have seminar talks and scientific discussions from December 27th, 2015 to January 17th, 2016.

I am looking forward to meeting you at our university. If you have any further questions, please do not hesitate to contact Dr. NGUYEN Thi Thu Van.

E-mail: ntrvan@hcmus.edu.vn  
Handphone: +84.932.534.197

Sincerely yours,

Prof. Dr. VU Hai Quan  
Vice president of VNUHCM- University of Science
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<td>Colombo, G.; Henrion R.; Hoang N.D.; Mordukhovich, B</td>
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CONGRESOS

N°: 1
Autor (a)(es/as): Nguyen Dinh Hoang
Título (Idioma original): Optimal Control Problems with State Constraints: The case of Maximal Monotone Differential Inclusions
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Ciudad: Hochiminh
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ANEXOS

A continuación se detallan los anexos físicos/papel que no se incluyen en el informe en formato PDF.