DISTRIBUTED PREDICTIVE CONTROL FOR FREQUENCY AND VOLTAGE
REGULATION IN MICROGRIDS

TESIS PARA OPTAR AL GRADO DE
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El control secundario distribuido en una micro-red requiere del intercambio de información entre controladores vecinos; por lo tanto, el desempeño general de la micro-red se ve afectado por la red de comunicaciones.

Esta tesis propone dos esquemas de control secundario predictivo distribuido, donde los modelos de predicción propuestos utilizan las ecuaciones droop y de transferencia de potencia de cada generador, de igual forma, la latencia y conectividad del canal son considerados, permitiendo una alta tolerancia a fallas eléctricas y de comunicaciones.

El primer esquema propuesto considera la regulación de frecuencia en la micro-red y el consenso de potencia activa en la micro-red. El segundo esquema propuesto, extiende el alcance, incluyendo la regulación de tensión y el consenso de potencia reactiva, entendiendo la co-dependencia de estas variables. Ambos esquemas propuestos incluyen restricciones operativas para asegurar soluciones factibles en la optimización.

Los resultados experimentales muestran que los esquemas propuestos (i) responden a los cambios de carga propios de la micro-red, trabajando en los rangos operativos; (ii) preserva los objetivos de control cuando alguno de los DGs es desconectado/reconectado a micro-red sin la actualización de parámetros del controlador; y (iii) compensa los efectos de la red de comunicaciones sobre la dinámica de la micro-red.
Abstract

Distributed secondary control in microgrids requires the information sharing among neighboring controllers; therefore, the microgrid performance is affected by the communication network phenomena.

This thesis proposes two distributed predictive controllers on the secondary control level of microgrids; where the proposed prediction models are based on droop and power transfer equations, but communication features such as connectivity and latency are also included, thus making the proposed controllers tolerant to electrical and communication failures.

The first proposed scheme is focused only on frequency regulation and active power consensus among the microgrid DGs, whereas the second approach adds voltage regulation and reactive power consensus as control objectives, regarding that all these variables are co-dependent in the microgrid. Both schemes include operative constraints in order to ensure the optimization feasibility.

The experimental results show that the proposed scheme (i) responds properly to load variations, working within operating constraints such as generation capacity and voltage range; (ii) preserves the control objectives when a power unit is disconnected and reconnected without any user updating in the controllers; and (iii) compensates the effects of communication issues over the microgrid dynamics.
To my Family, because without its support, patience and confidence, this work would be just a dream....

"Imagination will often carry us to worlds that never were, but without it we go nowhere."
-Carl Sagan
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Chapter 1

Introduction

1.1 Research Motivation

To take actions against the climate change promoting developments in renewable energies, and to ensure access to sustainable and reliable energy, especially to non-grid connected remote areas, have been included since 2015 by the Organization of the United Nations as two of the Sustainable Development Goals to achieve at year 2030 [13]. This has headed the academic and industrial communities to research about the integration of renewable energy in several scenarios, and proposing a new paradigm of electrical systems called smart grids. In [14, 15, 16], a set of concepts, researches and study cases are presented, being recognized by the academic community as a multidisciplinary effort which gather the progress in the smart grids paradigm.

One remarkable concept in the smart grid paradigm is the microgrid. It is a set of medium-voltage and low voltage distributed generators and loads, interconnected by transmission lines, protection and isolation devices, and power electronics converters, which acts as a single controllable entity [14]. Microgrids should be able to attend the next three issues:

- **Integration among different types of distributed generators (DG):** It is the capacity to integrate different types of energies such as photo-voltaic, eolic, etc, using power electronics converters as interface between each DG and the microgrid, and connect/disconnect them according to their availability.

- **Frequency and voltage regulation to ensure power quality in isolated microgrids:** When the microgrid has not an interconnection with the main grid, small changes in the power flow can produce significant deviations in frequency and voltage regulation, then the microgrid should compensate these disturbances to ensure power quality.

- **Connection to/ disconnection from/ coordination with/ either the main grid or other microgrids:** If it is available, the connection between the main and the micro grids is settled by the possible scenarios to transfer power between them, usually these scenarios include either economic constraints or demanded power constraints.

To manage these issues properly, the microgrid control has been stated as a hierarchical
structure [17, 18, 19], where each level is associated with a defined task. The primary level controls the local current and voltage over power electronics converters output, and gives statism to the microgrid, deviating the frequency and voltage when the demanded power changes. It is known as droop control. The secondary control level compensates the deviations and regulates the microgrid frequency and voltage and, in some cases, ensures the real and reactive power sharing through the DGs. The third control level optimises the dispatch according to a microgrid power and load forecasts.

Because of the second and third control levels acts over global microgrid variables, these require a communication network to share the DGs information and calculate properly their control actions. Therefore, the integration of electrical microgrid and communication network implies challenges related to the reliability, availability and flexibility of the system to ensure the real time operation and microgrid performance [20].

Using internet of things (IoT) based protocols is, nowadays, the best way to integrate devices such as smart meters, phasor measurement units (PMUs), and controllers to the microgrid, instead of typical protocols such as EtherCat, Modbus, or power line communication protocol (PLC). It is because these protocols were designed to a client-server topology, and it implies either a centralized control scheme, or a bottleneck of data packets when a large amount of DGs are integrated with a distributed control scheme. In both cases, if the central controller or the communication server fails, the microgrid performance is seriously affected. IoT allows direct communication between devices using a machine to machine scheme (M2M), and to use of virtual structures with a dynamic reconfiguration of the communication network, it is called software defined networking (SDN) and it is ideal to implement decentralized control schemes, improving the microgrid flexibility [20].

However, issues such as latency, data dropouts or changes on the communication network topology are persistent even always that a communication network is used [21]. Latency refers to the delay related to the transmission/reception process between any two points in the network. Data dropouts is a phenomena related with either routing or traffic issues on the network. When routers or terminal devices cannot identify the data packet successfully, the communication protocol discards it, and in some cases, a re-transmission is required, increasing the data traffic on the network. Changes in communication network topology is a phenomena usually related to wireless channels, which are susceptible to obstructions on the line of sight among devices, weather phenomena or interference from other wireless channels.

Moreover, to use technologies based on internet protocols (IP), increase the risk of cyber attacks over the microgrid infrastructure, therefore the control and communication technologies should have a proactive operation, able to isolate the attacked device and if it is required, disconnect it from the microgrid. This extends the flexibility requirement because the DGs or loads can be disconnected from the microgrid not only by electrical issues, but also by communication issues.

Communication issues are more significant when real time operation is desired. It means that the secondary control level, which is executed in seconds, is more susceptible to be affected by these issues than the tertiary control level, which is executed in minutes or hours. Then, the bellow challenges can be stated for secondary controllers in microgrids:
• **Frequency and voltage regulation:** These are the main tasks of secondary controllers. Frequency and voltage deviations related to the primary control response should be compensated, considering the synchronization and microgrid stability when DGs or loads are connected to or disconnected from the microgrid.

• **Plug and play (PnP) capability:** Because both electrical layer of the microgrid, and the communication network have flexible topologies, secondary controllers should be able to operate under these scenarios, updating the information used to compute the control actions.

• **Distributed scheme:** Distributed secondary controllers reduce the bottle-neck issue on the communication network used to exchange the information, because this type of schemes can be solved using only information from closest (neighboring) DGs. Additionally, this type of schemes avoid issues such as critical single point failures, or extended microgrid models, related with centralized controllers.

• **Latency and data dropouts compensation:** Because the information exchange among microgrid devices (i.e. Controllers or smart meters) requires a communication network, these issues should be considered by secondary controllers to improve the microgrid performance.

• **Execution time:** Secondary controllers should carry the microgrid to its steady state in a period of seconds, then the execution period needs to be less than the settling time in order to act properly against disturbances.

These challenges have headed the microgrid secondary control research in last years, however to design controllers covering two or more of these challenges simultaneously is an emergent topic in the area. Therefore, controllers designed to non-ideal conditions allow to set the feasibility of self-managed microgrids, specially on remote areas where operation and maintenance tasks are not easily attended.

### 1.2 Problem Statement

Networked control is the area which studies the relationship between control and communication systems, and its researches have been applied in the microgrid context. One widely-accepted topic from networked control used on secondary controllers on microgrids is the consensus problem, which is a distributed scheme based on a graph representation of the communication network topology, called adjacency matrix, to achieve a common goal among the DGs [22].

In [3] the consensus is inducted to achieve a precise real and reactive sharing among non-identical DGs. It is known as distributed proportional-integral averaging controller (DAPI), and it adds a term to each one of the proportional-integral (PI) controllers used to regulate the frequency and voltage on the microgrid. The DAPI controller is considered to be a PnP controller because the adjacency matrix can be updated online, and then the control law changes if a DG is connected to or disconnected from the microgrid. Thus, consensus is a suitable way to attend the changes in the communication network topology, however it is not
able to compensate the effects of latency or data dropouts over the microgrid performance.

Model-based predictive control (MPC) is a proper technique to compensate latency and dropouts in control systems, but it has not been totally exploited as a solution in secondary control level of microgrids. The most significant issue to use MPC in secondary controllers is the considerable computational effort required to solve the optimization problem, therefore the reported applications are focused on simulate unconstrained optimization problems using centralized and distributed schemes, and allowing in some cases, an analytical solution.

In [5] a centralized MPC for latency compensation is proposed for frequency regulation in a microgrid with two DGs; whereas in [23] distributed MPC with consensus is proposed for voltage regulation. In both cases, the proposed controllers show a good performance against the latency effect; but just the distributed approach is capable to face changes in the communication network topology without compromise the regulation task.

In a distributed model-based predictive control (DMPC) scheme, a (discrete-time) system model is used by each controller to predict its self-behavior over a prediction horizon. The used model is based on local information (i.e., measurements) and shared information from other controllers (i.e., previously computed predictions). The system solution minimizes a cost function based on the predicted trajectory and the information exchanged with other DGs. Although the optimal solution provides a sequence of control actions, only the first element is applied, and the optimization problem is solved again at the next sampling period (rolling horizon scheme) [24].

This thesis presents a novel approach of DMPC applied on the secondary level on microgrids. This approach is based on a constrained model which allows a numerical solution of the optimization problem with an adequate computational effort. The used model for prediction purposes is local for each DG in the microgrid, and it considers equality and inequality constraints, based on phenomenological and operative requirements. The model is built from droop and power transfer equations, but communication features such as connectivity and latency, and operational features such as power capacity and acceptable voltage range are also included. Therefore the proposed controller is tolerant to electrical and communication failures.

Because it is usual that DGs are connected to the microgrid using coupling inductances, is possible to state power transfer equations to estimate the contribution of $DG_i$ to the microgrid without a whole microgrid model on each controller. To do it, voltage, phase angle and frequency real-time measurements and estimations should be taken up ($V_i, \omega_i, \theta_i$) and down ($V^*_i, \omega^*_i, \theta^*_i$) stream of the coupling inductance $L_i$, as it is shown in Fig. 1.1. These values are used to update the local model for each controller once per sampling period in addition with the information shared from other DGs.

The DMPC optimization uses a cost function composed of six terms which represent the control objectives. Two of them regulates the average frequency and average voltage. Other two terms are related with the consensus over the contribution of real and reactive power accordingly with the capacity of each DG on the microgrid. These regulation and consensus terms are global control objectives on the microgrid, and to achieve these, the information exchange among DGs is required. Finally, controller outputs $\Delta \omega_{s,i}$ and $\Delta V_{s,i}$ are calculated
as optimization variables. Although the optimization gives a control sequence as output, just the first value is used to compensate frequency and voltage deviations.

The output vector $X_i$ contains the information from $DG_i$ to be sent to the units with a direct communication path, which are called neighbors. Whereas the vector $X_{ij}$ is the information vector received on $DG_i$ from $DG_j$, which is subjected to connectivity and latency, represented by $a_{ij}$ and $\tau_{ij}$ respectively. Vector $X_{ij}$ is composed of predicted values of frequency, voltage, real and reactive power required as optimization problem inputs, and therefore, required to estimate $X_i$ at the next iteration. Because this vector includes estimated future information, it is used to compensate latency and data dropouts effects.

![Figure 1.1: Proposed DMPC scheme for $DG_i$](image)

Hence, the proposed DMPC supports the PnP requirement on electrical and communication scenarios, also, it compensates latency and dropouts effects using the predicted variables, and it allows the inclusion of operational constraints as power capacity and voltage range, limiting the set of feasible solutions and reducing the computational effort.

### 1.3 Hypotheses

The hypotheses related with this work are:

1. Stating a local model for each DG on the microgrid, avoiding a whole microgrid model, allows to include operational constraints in the DMPC formulation, without compromise neither frequency regulation, nor active power consensus.

2. Including into the optimization problem a communication model for latency and connectivity, adds plug and play capability and delay compensation to the DMPC, allowing to face short term phenomena such as data dropouts or latency, and long term phenomena such as changes on electrical or communication topology, without any parameters updating by external user.

3. Considering the co-dependent relationship among frequency, voltage, active and reactive power on each DG, it is possible to design one controller to achieve global regulation
and consensus objectives simultaneously in an heterogeneous microgrid, instead of two controllers as is often reported.

4. Building properly the optimization problem, and updating it with real-time measurements, it is possible to use numerical methods in a real time controller, achieving a feasible operation even when strong disturbances as large changes on demanded or supplied power occur.

1.4 Objectives

1.4.1 General Objective
To develop novel DMPC schemes for secondary control level of microgrids, with plug and play capability, able to operate when both electrical and communication failures occur.

1.4.2 Specific Objectives
1. To design a DPMC using a local electrical model and information shared from neighboring DGs, to achieve frequency regulation, and consensus over the active power contribution among the DGs on the microgrid, considering the settling time required for secondary controllers.

2. To synthesize a communication model, and include it into the DMPC formulation, in order to preserve the microgrid performance (overshoot and settling time), against the latency, data dropouts and changes on communication and electrical topologies.

3. To propose a novel model for DMPC purposes, which merges the co-dependent relationships among frequency, voltage, active and reactive power, and including electrical constrains to bound the feasible solution space.

4. To validate the DMPC real-time operation using an experimental setup, under scenarios with both electrical and communication failures.

1.5 Contributions
This thesis proposes a novel DMPC scheme applied to the secondary control level on microgrids. The proposed controller considers as control objectives frequency and voltage regulation and consensus over the real and reactive power contributions from each power unit in the microgrid. The used model for prediction purposes is based on droop and power transfer equations, but communication features such as connectivity and latency are also included. The local model on each controller is updated once per sampling period with local measurements over each DG and with information shared from neighboring DGs. The model permits including explicit operational constraints, such as voltage range and apparent power limits. The main contributions of this work are listed bellow:

- **The proposed model joins control objectives that usually are covered by two controllers.** To use of co-dependent equations for frequency, voltage and power allows a
proper prediction of these variables, and to calculate secondary control actions using just one optimization problem.

- **The proposed model is distributed and PnP.** To use local models for electrical and communication phenomena, avoids a whole electrical model of the microgrid, required to achieve global microgrid goals such as voltage or frequency regulation, even when both, DGs and loads are connected to or disconnected from the microgrid, and compensates issues such as latency and data dropouts as well.

- **The proposed DMPC is implementable.** To use phenomenological and operational constraints, bounding the feasible solution space, reducing the computational effort, and achieving the regulation and consensus goals, even when strong disturbances such as both electrical and communications topologies change simultaneously.

The aforementioned contributions where validated by the microgrids community in the published paper:


The published paper presents the theoretical framework, simulated and experimental results using heterogeneous microgrids. The experimental setup was built in the Microgrids Control Lab of University of Chile, using a three DGs microgrid; and it was also used to test other distributed control schemes, as was published in:


### 1.6 Thesis Outline

This thesis is organized as follows. In Chapter 2 the state-of-the-art about secondary control in microgrids, as well as remarkable topics about communications in microgrids, are discussed. A first DMPC approach for frequency regulation and active power consensus is proposed in Chapter 3 introducing the model synthesis and quadratic problem formulation. In Chapter 4 the reactive power consensus and voltage regulation are including, extending the model proposed. Several simulation and experimental test to prove the DMPC capabilities are also included in this chapter. Finally, the conclusions and final remarks are presented in Chapter 5.

To facilitate the reading process, a list of acronyms is included in appendix A1 whereas mathematical procedures used along this thesis are shown in appendixes A2 to A4. A description of Triphase® platform used on the experimental setup is presented in appendix A5.
Chapter 2

Literature Review

This chapter presents the state-of-the-art of the main topics treated through this thesis. First, the main context is introduced, exploring the hierarchical structure used to control the microgrids and, presenting concepts such as reliability, flexibility and stability requirements for microgrids. Next, remarkable control schemes used for secondary level in microgrids will be shown highlighting the advantages and disadvantages of these; secondary controllers based on predictive control will take an special significance in this subsection. Finally, the main challenges in secondary control of microgrids are discussed.

2.1 Microgrids Framework

An electrical microgrid, or only microgrid, is defined as a set of distributed generators (DGs) and distributed loads, interconnected among them, using transmission lines, protection and isolation devices and, power electronics converters which acts as a single controlable entity [1]. However, external factors such as the weather or location and, internal factors such as power capacity, power demanded, or communications, produce that the microgrid to be exposed to changes in a permanent way. Therefore, the microgrids control community agrees to use an hierarchical structure where each control level is focused on issues according with an specific time span [10].

Standard IEEE 2030.7 [1] defines a control functions assignment on four blocks, which are commonly referred as control levels in literature, also defines that these can be grouped into device controllers, plant controllers and microgrid controllers, accordingly with the microgrid design and, hardware and software platforms used for implementation purposes. The control hierarchy and the task grouping options used for microgrids are shown in Fig. 2.1.

In the primary control level, the voltage and current inner loops are included. These act directly over the power electronics inverter and their configuration determine if the DG acts as a voltage source (VSI), as a current source (CSI) or if the DG uses a maximum power point tracker (MPPT). To achieve this, inner loops should respond in micro or milli-seconds. Furthermore, the primary control level also includes the droop control loops, which give statism to the microgrid, changing, in tens or hundreds of milliseconds, the voltage or
frequency operation point when the demanded active or reactive power change. The main purpose to use droop control in microgrids is that the DG dynamics on the microgrid acts in a similar way than a synchronous machine in a power system [25, 26].

The secondary control level compensates, in seconds, the frequency and voltage deviations caused by the droop controllers and therefore, the microgrid can remains in its nominal operation point. Although the main goal of secondary level is the frequency and voltage regulation, an additional function for this control level is to achieve an equitable power sharing among the DGs, improving the power supply and balance stability [27]. Unlike the primary control level, where the controllers are local, the secondary level requires information from several points of the microgrid, specially for voltage regulation and power sharing purposes.

An specification about the requirements of frequency and voltage regulation, active and reactive power sharing, frequency and voltage droop which are included into primary and secondary levels are shown in section 5.3 of [28].

The third control level is considered as a management and coordination level [17], where an optimization problem is solved to minimize the operation cost. In this level, topics such as the generation cost for each DG, the weather and demanded power forecasts or, the batteries state of charge and health are considered to take decisions about the microgrid operation in a period of minutes or hours.

Because the second and third control levels require a communication network, new disturbances, such as latency or data dropouts, are added to the microgrid [29]. Depending on how the controllers in these levels are interconnected, the communication network requirements are established. In a centralized control scheme, only one controller receives the information from all DGs and computes the control action required; in a decentralized scheme there exist one controller per DG and the control action is computed as a function only of local information; whereas in a distributed control scheme, the information is shared among controllers in order to compute the local controller output considering the global microgrid behavior.
Although a centralized scheme was an initial solution, the technical advances have allowed the evolution to distributed secondary controllers in microgrids. In Fig. 2.2, the structure for centralized and distributed secondary controllers is shown, whereas communication topics on microgrids will be discussed on section 2.2.

Figure 2.2: Microgrids Centralized and Distributed Secondary Schemes - Based on [2].

### 2.2 Communication in Microgrids

Communication network is a mandatory requirement in microgrids, however, the discussion about the impact in the microgrids control performance, of the communication issues such as latency, data dropouts, network topology, communication technologies, etc, is an emergent topic. In [7, 30, 8, 31] the communications protocols and technologies are discussed mainly under the Internet of Things (IoT) framework. IoT involves multiple concepts along the communications infrastructure, for example, although the IoT physical layer implies wireless communication, the used devices can operate at different frequencies (e.g. 2.4 GHz and 900 MHz for Zigbee and Z-wave technologies respectively); however not all of these concepts can be applied in the secondary control level of microgrids. In this context, machine-to-machine (M2M) communication, efficiency, compatibility, redundancy, privacy and security are relevant topics, whereas, backbone channels, cloud-based platforms or data logging are more compatible with the tertiary control level. At the next, remarkable concepts for communications on secondary control level of microgrids are discussed:

#### 2.2.1 Physical Channels

Although the IoT paradigm implies to use wireless technologies in order to exploit features such as scalability and coverage, wired technologies (e.g. cooper wires and optical fiber) are also used in microgrids control, remarking that these latter are less susceptible to interference issues than wireless channels [8].

There are three wired technologies used for data transmission: optical fiber, cooper based twisted pairs, and cooper based power lines. Optical fiber and unshielded/shielded cooper twisted pairs (UTP/STP) are often used in data networks, therefore, there are several devices and resources able to implement, operate and diagnose these type of networks. Moreover,
power line communication is a restricted technology because to scale this type of network, beyond of indoor environments, is more expensive than other wired technologies, mainly because data frames cannot be propagated through transformers [32].

Implementation and maintenance costs impact the cost-benefit ratio for optical and twisted pair networks because these technologies require dedicated ducts and connectors to ensure a good performance, whereas power line communication modulates and sends the data frames through the power lines, reducing the implementation and operation costs. The main disadvantage for UTP/STP networks is the reduced coverage (100 m) per wire section when transmission control protocol-internet protocol (TCP-IP) is used. Though using protocols such as RS232 or RS485 protocols, the coverage distance can be increased, serial protocols introduce compatibility and failures diagnosis issues to the networks.

Wireless channels are more versatile than the wired ones. Although 3G to 5G cellular, licensed radio frequencies, and microwaves technologies are robust solutions, to use third party facilities implies an expensive operation cost [9]; therefore, channels such as Wi-Fi, WiMax, ZigBee and LoRa have a better cost-benefit ratio. However, because these wireless technologies use unlicensed frequencies, there are more susceptible to interference, line of sight and security issues, which implies a high variance in latency and data dropouts [33]. As well as wired channels, the coverage depends on the used technology and, it can be from few meters, using Bluetooth technology, to few kilometers, using LoRa or Sigfox channels. An additional issue on wireless networks is the penetration capability. Channels with a lower frequency of operation will have a higher penetration, which is a remarkable capability when the microgrid operates in high obstructed environments, e.g. mountains or cities.

In Table 2.1 the most relevant types of physical channels in communication networks are summarized.

The communication network topology, as well as the technology required to implement it, are important topics in the microgrid design, and these depend on factors such as the economical investment and the geographical location of the microgrid. Although wired and wireless channels can be used in the same network, it is important to consider bottle-necks and compatibility issues. With these concerns, different protocols have been developed, and these can be categorized in networking and automation protocols.

### 2.2.2 Communication Protocols

**Networking protocols** consider the algorithms related with the routing, segmentation and signaling of data-frames. These protocols are implemented over TCP-IP networks and are required to improve the reliability, security and interoperability of communication networks and, these have not a direct relationship with the microgrid information (measurements, control signals, etc). According with the OSI/ISO reference model, networking protocols are associated with the layers one to six. To this category belong concepts such as IPV4 and IPV6, which are two approaches to identify the devices in the network; the virtual private networks (VPN) and quality of service (QoS), which are related with the data traffic management, also belong to this category [20].

**Automation protocols** are used to manage the process information, for this reason are
**Table 2.1: Communication Technologies Applicable in Microgrids [7, 8, 9, 10, 11, 12]**

<table>
<thead>
<tr>
<th>Type</th>
<th>Technology</th>
<th>Max Coverage</th>
<th>Rate</th>
<th>Frequency Band</th>
<th>Advantages</th>
<th>Disadvantages</th>
<th>Suitability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wired</td>
<td>Optical Fiber</td>
<td>60 Km</td>
<td>10 Mbps (IEEE802.3j) to 100 Gbps (IEEE802.3cd)</td>
<td>Near-infrared and visual spectrum</td>
<td>-High speed -Low Interference -Low Latency</td>
<td>-Expensive Technology -Complex scalability -Expensive maintenance</td>
<td>Primary and secondary control</td>
</tr>
<tr>
<td>Twisted Pairs</td>
<td></td>
<td>100 m</td>
<td>10 Mbps (IEEE802.3i) to 10 Gbps (IEEE802.3an)</td>
<td>&lt;25 MHz to &lt;500 MHz</td>
<td>-Easy access -Easy configuration</td>
<td>-More capabilities-more cost -Complex scalability (outdoor)</td>
<td>Primary and secondary control</td>
</tr>
<tr>
<td>Power Line</td>
<td></td>
<td>200 m</td>
<td>14-200 Mbps (IEEE 1901) to 10 Gbps (IEEE802.11ac)</td>
<td>&lt;100 MHz</td>
<td>-Low cost implementation</td>
<td>-Fixed topology (non-flexible) -Complex scalability (outdoor)</td>
<td>Tertiary control</td>
</tr>
<tr>
<td>Wireless</td>
<td>Wi-Fi</td>
<td>100 m</td>
<td>2 Mbps (IEEE802.11) to 2.4 GHz</td>
<td>-Cheaper technology -Easy configuration -Easy scalability</td>
<td>-Susceptible to interference -More security-more cost</td>
<td>Secondary control</td>
<td></td>
</tr>
<tr>
<td>LoRaWan</td>
<td></td>
<td>5 Km (urban), 20 km (rural)</td>
<td>&lt;50 kbps</td>
<td>433/ 868/ 915 MHz</td>
<td>-Low power consumption -High penetration -Easy scalability</td>
<td>-Low speed -Restricted downlink</td>
<td>Tertiary control</td>
</tr>
</tbody>
</table>

usually associated with the seventh, or application, layer of OSI/ISO reference model, despite some protocols can specify some requirements of the low layers. Although protocols such as MODBUS, Profinet and EtherCat are applicable for several types of process, there are dedicated protocols to electrical systems, such as DNP3 and IEC61850. The relevance of automation protocols is that they define the data flow in the network, then are related with the control scheme implemented in the microgrid. For example, microgrids with several automation protocols implies to use of a centralized client-server scheme, where the server has the "translator" role (OPC servers are used with this purpose). For secondary control level, distributed, or at least multi-master multi-slave protocols, which allow machine-to-machine communication, are suggested; DNP3 and IEC61850 satisfy this requirement, and, because these are designed for electrical systems, have a higher performance reporting failures or events. Other schemes such as device-to-cloud or device-to-gateway are required to centralized tertiary controllers, where the load and generation profiles are used with forecasting purposes [20, 34].

Using communication protocols for specific purposes such as data encryption or security services to protect the data integrity, QoS to ensure a predefined band-width on multipurpose networks, GPS synchronization or multi-master multi-slave schemes to bring M2M communication, implies a higher data traffic over physical channels, then latency, data dropouts and data-links availability will have a higher impact in the control performance.
2.2.3 Communication Issues

As it was explained, communication technologies and protocols carry issues that can affect the control performance in the microgrid. Although these issues have several causes, all of these can be summarized on two effects: latency and data dropouts.

**Latency** is defined as the time delay associated to the transmission and reception of data-frames between two devices in the network. For control purposes, latency has an end-end approach; it means that the time required by the communication and automation protocols, as well as the time required to process the information as a controller input, are merged. The latency effect is related with the sampling period of the control system. Low latency values on control systems are between tens and hundreds of milliseconds; being useless to use communications networks for primary control level on microgrids. In [34] is stated the relevance of the worst-case latency instead the average latency in industrial networks, being used as a valuable tool on network calculus.

Latency is correlated with internal issues, such as data traffic, routing algorithms and channel capacity; as well as external issues on the communication network, such as weather, line of sight and interference. Therefore, wired networks have less latency than the wireless ones.

**Data dropouts** represent the data frame losses in the network. As well as latency, data dropouts have several root causes, some of these are also related with latency. Non-optimal routing algorithms can produce data disorder and data re-transmissions, overcharging the network and discarding data frames. Moreover, external issues such as line of sight obstructions, defective physical connections or defective synchronization can increase the amount of spontaneously lost data-frames or even can produce the disconnection of the communication link, affecting the network topology [29].

To include the effects of latency and data dropouts in the development of controllers and in its performance analysis is an implicit task on the secondary control level of microgrids, because the sampling period of these controllers and the effects of these communication issues are in the same time range. The multiple possible architectures for communication networks allow several control schemes for secondary level. The most remarkable of these are discussed at the next section.

2.3 Secondary Control in Microgrids

As it was mentioned, the main function of secondary controllers is to restore the microgrid frequency and voltage to the nominal values \((\omega_0, V_0)\), after disturbances such as load impacts or changes in the generation capacity. Assuming a classical approach, when the microgrid is disturbed, the primary control deviates, the operation point from the nominal point, according with the droop curves defined for the DG\(i\) as (2.1) and (2.2); where \(M_{\omega,i}, M_{qv,i}\) are the droop slopes and, \(P_i, Q_i\) the active and reactive power supplied by DG\(i\). The secondary restoration is done through \(\omega_{s,i}\) and \(V_{s,i}\) which displace vertically the droop curve, preserving the power supplied, until the frequency and voltage returns to their nominal values [35].

The operation of droop and secondary control after a disturbance on active power (load
impact) is shown in Fig. 2.3. Note that at $t = 1$ the microgrid operates at the nominal frequency; once the load changes, at $t = 2$, primary control level modifies the power contribution of $DG_1$ and $DG_2$ in order to supply the new power demand. Finally, at $t = 3$ secondary control action, $\omega_s$, moves vertically the droop curves in order to restore the microgrid frequency, preserving the power contribution in both DGs.

\[
\omega_i(t) = \omega_0 + M_{\omega,i} P_i(t) + \omega_{s,i}(t) \quad (2.1)
\]

\[
V_i(t) = V_0 + M_{q,i} Q_i(t) + V_{s,i}(t) \quad (2.2)
\]

Figure 2.3: Droop and Secondary Responses Against Load Change.

Several secondary control schemes have been reported in specialized literature, which can be classified as centralized and distributed controllers [16]. Centralized schemes consist of one secondary controller which computes its action considering the information received from the DGs on the microgrid. Although central controllers are suitable for small microgrids and, it is possible to include redundancy and fault tolerance, to merge the microgrid information in only one point is a disadvantage of this type of controllers; additionally, it is not always possible to implement a star-topology communication network, as is required for this type of control scheme.

On the other hand, distributed controllers split the global restoration of frequency and voltage into local subproblems, relaxing the communication topology requirement and reducing the computing effort. In this case, the control action is computed locally based on the information exchanged with neighboring DGs. However, a network where one DG shares information with the all other ones in the microgrid (fully connected network) implies a high bandwidth to ensure the performance required.

At the next, reported centralized and distributed secondary controllers are detailed.
2.3.1 Centralized Secondary Control in Microgrids

In [5], [2] and [36] centralized secondary controllers are proposed for comparison purposes when communication issues such as latency or changes in network topology disturb the microgrid.

In [5] a typical proportional-integral (PI) controller, a PI controller with a Smith predictor and an MPC, used for frequency regulation, are compared, using a centralized approach, in a microgrid with two DGs. The comparison is based on the microgrid performance when the latency in the microgrid increases. Although the Smith predictor is a technique for latency compensation in systems with PI controllers, stability issues are evidenced when the latency is greater than the tuned delay in the predictor. Then, as a result, the MPC scheme shows the best performance in this comparison, considering its stable behavior when the latency increases.

Another way to compensate the latency using a PI central controller is shown in [37], where the controller gain is scheduled according with a delay estimation based on the time stamp of the received data. The gain scheduling criteria used in this case is based on the estimation of the system poles considering the communication delay.

In [2] and [36], PI based controllers are used for latency and topology changes compensation. The comparison is based on the microgrid performance using centralized and distributed controllers. In [2], the microgrid is disturbed increasing the latency and data dropouts, whereas in [36] the communication network topology is included in the distributed scheme. In both cases distributed schemes present better performance than the centralized ones, and their results will be discussed on section 2.3.2.

In conclusion, these works evidence that the inclusion of communication phenomena in the secondary controllers improves the microgrid performance in real environments. The stability analysis developed on these papers show a relation between the system robustness and an updated model of the communication network, implying an incorrect microgrid behavior when the communication parameters estimation fails. Also, these papers show that the frequency regulation using a centralized scheme is feasible using only one measurement point, it is because the frequency is an universal parameter in the whole system, not so the voltage regulation, which requires different measurement points, because the line impedance and reactive power flow allows different voltages in the microgrid.

2.3.2 Distributed Secondary Control in Microgrids

From [2] and [36] two facts can be stated: (i) Power sharing can be improved in the microgrid including control actions related with the active and/or reactive power in the secondary level and, (ii) Because distributed schemes require information exchange among the DGs on the microgrid, the communication topology should be included in the secondary controller design. Although issues such as latency and data dropouts also impact the microgrid behavior, the scientific community accepts that a distributed secondary control level is inherently fault tolerant to electrical issues [38].

A remarkable communication issue in secondary level is the network topology variability.
A simple way to adapt the control law when a DG is disconnected from or reconnected to the microgrid is using the adjacency matrix, which was introduced in [36]. The adjacency matrix is a graph-based representation of the information flow among the DGs, where the graph vertices represent the DGs and the graph edges represent the communication links [39]. This matrix permits to use properties of the network systems in the microgrids context to analyze the impact of some communication phenomena in the system stability.

Although several schemes, that use the adjacency matrix, have been reported [40, 41]; the most representative controller in this context is the distributed average proportional-integral (DAPI) controller, which combines the simplicity of PI control with a power consensus problem, in order to lead the same real and reactive power contribution for each DG in the microgrid. In [3] the DAPI controller for reactive power and voltage regulation is introduced, however it is demonstrated that using this control scheme in an heterogeneous microgrid, there is a trade-off between the voltage regulation and reactive power consensus. It means that it is not possible that the DGs have the same reactive power contribution if is also desired that all DGs operate at the same voltage.

The control law for DAPI controllers used in [3] are shown in (2.3) and (2.4), where $K_{i,\omega}$ and $K_{i,v}$ are the controllers gain, $Q^*_i$ and $Q^*_j$ are the reactive power ratings, $a_{ij}$ is the adjacency term, which is equal to one if the communication between DG$_i$ and DG$_j$ is stated and zero otherwise and, $\beta_i$ and $b_{ij}$ are coefficients related with trade-off between the voltage regulation and reactive power consensus. Fig. 2.4 presents a diagram of DAPI control scheme, where $d_i = \sum_{j=1}^{p} a_{ij},$ and $\delta_i = \sum_{j=1}^{p} b_{ij}.$

$$K_{i,\omega} \frac{d\omega_{s,i}}{dt} = -(\omega_i - \omega_0) - \sum_{j=1, j \neq i}^{p} a_{ij} (\omega_{s,i} - \omega_{s,j})$$ (2.3)

$$K_{i,v} \frac{dV_{s,i}}{dt} = -\beta_i (V_i - V_0) - \sum_{j=1, j \neq i}^{p} b_{ij} \left( \frac{Q_i}{Q_i^*} - \frac{Q_j}{Q_j^*} \right)$$ (2.4)

From (2.3), it is possible to state that in the steady state the derivative term is equal to zero, then the local frequency $\omega_i$ achieves its nominal value $\omega_0$ and furthermore the control actions $\omega_s$ are in consensus, implying at the same time, maintaining the real power sharing achieved by the droop control as it is demonstrated in [3].

In an effort to compensate the latency and, at the same time to consider the communication topology, the finite time control scheme was introduced to the microgrids context. In [12] this control technique was presented as a solution which ensures the convergence of the frequency regulation and active power consensus in a finite time. Although in [12] the communication latency is not included, extensions of this control technique, such as [13, 4, 44] and [45] include it and analyse the microgrid performance in at least four scenarios (i) load impacts, (ii) changes in the communication topology, (iii) latency changes and, (iv) plug and play capability.

All of these works use similar control laws for proposed controllers, including consensus for power sharing; however, a remarkable topic for finite time controllers is that these also apply
consensus for voltage and frequency regulation, instead to local terms as DAPI controllers. This allows to assign the leader role for one DG on the microgrid. Also, in some cases such as [43] and [44], the non-linear sign function is included to the consensus terms, in order to switch the controller gain according with the consensus error and turn these term to zero when the error is null.

Fig. 2.5 shows the finite time control scheme used in [4]. In this case each control objective is managed by one independent controller, implying four controllers instead of two as the DAPI scheme.

Another promising approach to manage simultaneous communication issues, and at the same time optimize the microgrid performance, is using model-based predictive control (MPC) [46]. MPC allows analytical implementations when the optimization problems are unconstrained, as well as the inclusion of operational constraints, such as power or voltage ranges for each DG, when numerical solutions are used; therefore, to reduce the computational effort in order to a real-time implementation is a challenge. At the next section DMPC schemes reported for microgrids secondary control are discussed.

### 2.3.3 Secondary Control Based on DMPC

In the microgrids context, predictive control is usually applied in tertiary level to optimize the power dispatch and the load management in the microgrid [47, 48]; also, some applications of MPC have been reported for primary control to mitigate harmonics or unbalance issues over power electronics converters [49, 50]. However, the distributed application for tertiary, and
superior levels, is an emergent topic \[51\], whereas for primary level the reported applications are totally decentralized (without communication among controllers).

In a distributed model-based predictive control (DMPC) scheme, a (discrete-time) system model is used as a set of (equality and inequality) constraints by each controller to solve an optimization problem and to predict its self-behavior over a prediction horizon. The model used is based on local information (i.e., measurements) and shared information from other controllers (i.e., previously computed predictions). The system solution minimizes a cost function based on operational and/or economical requirements. Although the optimal solution provides the sequence of control actions through the control horizon, only the first element is applied, and the optimization problem is solved again at the next sampling period; it is called as the rolling horizon scheme \[24\].

Several DMPC approaches have been reported in literature, not only on the microgrids context, and these can be classified according with parameters such as optimality, topology, robustness, etc; however, two categories should be highlighted \[52, 53, 54\]:

- **According with the cost function**: DMPC can be defined as a cooperative scheme if the same global cost function is used for each distributed controller, whereas non-cooperative schemes show coupled variables only in the problem constraints. Both schemes require the information sharing among controllers, and it implies that should be assumed that the real system states belong to the neighborhood of the predicted states. Non-cooperative schemes can be favorable for systems with dynamic structure, such as plug and play systems; on the other hand, cooperative schemes are adequate for systems that require coordination among controllers.

- **According with the computation type**: For non-iterative methods, the problem inputs are updated and the solution is computed once per sampling period, then, at
the end of this, the outputs are updated and shared to the other controllers. Iterative methods exchange information and solve the optimization as many times (iterations) per sampling period as needed until achieve a terminal condition. At the end of each sampling period, the controllers update their outputs. In this approach, the measurements are preserved as constants along of the iterations. Non-iterative methods are used as solutions where the computation time and/or communication requirements are critical issues, whereas iterative methods are implemented when a global optimal solution is required.

Another approach reported is the sequential approach, where the controller $i$ solves its optimization using the information received from the controller $i - 1$. Once the problem is solved and the control action applied, the predicted states are sent to the controller $i + 1$. Sequential approach is similar to the token ring scheme for communication networks, where the user which has the token access to the channel. In [55] the sequential approach is applied to a consensus problem, however, it is a remarkable issue that in this type of schemes the settling time of the whole system increases if the number of controllers increases too.

Because the computational effort required to solve the optimization in hundreds of milliseconds, and to state a feasible problem even the electrical and communication disturbances in the microgrid, there are not several applications of DMPC in the secondary level of microgrids. The most of reported schemes in this context, are based on unconstrained optimization problems, where analytical solutions can be derived, therefore, the computational requirements to implement these are reduced, whereas methods which require numerical solutions are usually validated through simulations. At the next, reported DMPC schemes for secondary control level are discussed.

A DMPC for voltage regulation, including consensus, has been proposed in [56]. The analytical solution of the optimization problem is derived from a linearized state space model of the primary control level (inner-PI and droop controllers). An extension of this scheme, covering communications issues, is reported in [23, 57]. In this case the information shared among the controllers in the microgrid, consist of the linearized state vector, which is required to update the control law of neighbor controllers. A disadvantage of this scheme is that operational constraints can not be included, therefore, non-feasible solutions can be admitted. In both cases, the frequency regulation is managed through a DAPI controller.

Other non-iterative DMPC schemes are proposed in [58, 59]. These both controllers are based on a local model of the whole microgrid which is used locally in order to solve the optimization. In [58] the voltage regulation is achieved using a local estimator which computes the node voltages for the whole microgrid, whereas the local DMPC uses these estimations to solve the optimization problem. In [59], the unconstrained optimization is solved analytically using an equivalent transfer function of the whole microgrid. In this case, independent controllers are designed for frequency and voltage regulation without power consensus. Although typical microgrid disturbances such as load changes are compensated, these schemes are sensible to topological issues (DG connections or disconnections) due the use of the voltages estimator.

In [60], a decentralized scheme for frequency and voltage regulation is proposed, consid-
ering the co-dependent behavior of these variables. In this case two optimization problems are formulated; one of these to control PV generators, and the second one to control storage units in the microgrid. Both optimization problems consider phenomenological models where voltage and frequency are coupled variables included in the state equation, which is used as a set of equality constraints. Although the simulation results show a good performance for regulation task, the non-communication among controllers does not allow a cooperative managing of active and reactive power.

A similar approach, where the DMPC controller is designed considering the model generator is presented in [61]. In this iterative scheme a constrained optimization problem is built using, as equality constraints, a prediction model which assumes the generators model (synchronous generators and wind turbines) and, voltage ranges as inequality constraints. Although experimental results (using a two DGs microgrid) and simulation results (using a six DGs microgrid) validate the control performance, strong assumptions such as an ideal communication network and, fixed microgrid topology are used. These assumptions imply that a plug and play scenario and, latency, dropouts or communications topology are not considered.

In [62], an iterative DMPC frequency regulation and including optimal economic dispatch is proposed. Gas turbine, wind turbine, fuel cell, diesel and photo-voltaic generators, as well as a power demanded and power supplied forecaster, are assumed in the microgrid. Because the power contribution of each DG is settled according with the economic cost, the consensus is not included in this scheme. The computational complexity required to solve the optimization and, the information exchanged on each iteration, limit the experimental implementation of this scheme.

As it was shown several DMPC schemes are proposed in literature, however, the reduced amount of experimental results indicates that the computational effort required to deploy experimental solutions on real time systems is still a challenge. As well, it is remarkable that the predictive control advantages, such as delay compensation and the use of constraints, have not been totally exploited for secondary controllers in microgrids; even, capabilities accepted for the microgrids community, such as power consensus or plug and play schemes, that are included in non-predictive controllers, are not often found in predictive schemes.

In Table 2.2 the main control schemes reported for secondary level in microgrids are classified according to the approaches used.

2.4 Discussion

Considering the literature reviewed, it is possible to state that the analysis of communication phenomena over the control system dynamics is an emergent topic because new paradigms, such as IoT, have increased the amount of data through the industrial networks, affecting the control performance. Along the last decade, new control strategies which require to exchange information to take decisions have been reported. Therefore, concepts such as connectivity and availability, as well as the channels capacity should be taken into account when the control schemes are developed to operate on real time applications.
Table 2.2: Secondary Control Schemes Used in Microgrids

<table>
<thead>
<tr>
<th>References</th>
<th>Topology</th>
<th>Control Law</th>
<th>Control Objectives</th>
<th>Communication Issues</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Centralized</td>
<td>Decentralized</td>
<td>Distributed</td>
<td>PID</td>
<td>Finite Time</td>
</tr>
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<tr>
<td>PI</td>
<td></td>
<td></td>
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<td>X</td>
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<tr>
<td>Proposed DMPC</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Although secondary controllers for microgrids, which include frequency and voltage regulation, as well as power consensus, have been proposed and accepted for the community, solutions that consider the co-dependent behavior among frequency, voltage and active/reactive power are unusual. Furthermore, the fact to consider a communication model in these controllers, which at least considers the network topology, has demonstrated to improve the stability, reliability, and flexibility of the microgrids.

The two main advantages of linear control schemes, such as DAPI, are the reduced computational cost when are deployed, and the intuitiveness on their parameters. Although unconstrained predictive controllers can include these capabilities too, when the optimization problem requires more information to improve the microgrid performance, the computation and the communication requirements increase, turning these controllers slower and susceptible to communication phenomena.

DMPC is able to manage the control and the communication requirements simultaneously, however the trade-off between the optimization problem complexity and the sampling period required on the secondary control level should be stated according with microgrid specifications. The model used to predict the system trajectory should considers the microgrid heterogeneity (e.g. different power capacities) and the co-dependent behavior among the system variables. However, to use an electrical model of the whole microgrid on each distributed
controller restricts the plug and play, as well as the scalability capabilities. Therefore, the optimization problem used on these controllers should be based on local measurements and estimations of the local DG, as well as on information received from neighboring DGs, in this way, the flexibility and scalability are preserved on the microgrid. Note that these statements do not imply that DMPC schemes on secondary level should be neither cooperative or non-cooperative, nor iterative or non-iterative, because it depends on the control objectives stated on the optimization problem and how it is solved. However, the proposed controllers in this thesis are focused on cooperative and non-iterative approaches.

In Fig. 2.6 the taxonomic classification of the control schemes proposed in this thesis is illustrated. Note that this classification does not consider the type of model used, it means, if the controller uses a whole microgrid model as in [58], a model according to the generator as in [60], or an local electrical model as in this thesis. Also, note that the taxonomic classification is independent of the type of communication network and type of communication channel used to share information among controller; it is because just power line communication forces to the control scheme to use the same topology for electrical and communication networks.

![Figure 2.6: Taxonomic Classification of Proposed DMPC.](image)

According to Fig. 2.6, the controllers proposed in this thesis are classified as non-iterative DMPC, which use a numerical solver to compute the control action required to achieve the regulation and consensus objectives. Although [58] and [60] can be included in the same category, the main difference among these and the proposed controllers is the model used to solve the optimization. In this thesis, the model is based in local measurements and estimations, and complemented with previously predicted information from neighboring DGs, allowing to predict the local frequency, voltage, and active/reactive power and to compute the control action required to achieve the global control objectives.
Chapter 3

DMPC for Frequency Regulation

In this chapter a distributed model-based predictive control (DMPC) for frequency regulation and active power consensus is addressed using a quadratic programming (QP) structure. The proposed controller is based on a local model, using the droop and power transfer equations for prediction purposes. Delay and adjacency terms are included in order to face communication issues such as latency, and data dropouts. The optimization problem proposed considers information from neighboring DGs in equality constraint related with the average frequency estimation, and in cost function term related with the power consensus. Simulation tests shown a good microgrid performance against load impacts, and higher latency and unavailability of communication channels.

3.1 Model Used for Controller Design

3.1.1 Distributed Generator Model

In the specialized literature for secondary control design, an equivalent model based on a first-order transfer function, or based on a filtered response for droop control, is used as a valid approximation for the primary level response \[17, 1, 37, 59\]. Although the droop is the main dynamic in the primary response, to consider the power limits for each power unit is mandatory for microgrid implementations. Since this consideration establishes feasible solutions for the control action, the inclusion of an active power model on the secondary control level is desirable. In this case, the droop model should be extended using the active power transfer equation between the local power unit and the microgrid. The latter increases the complexity of the optimization problem.

Fig. 3.1 shows the control diagram for \(DG_i\) which operates as a voltage source; in this case the output filter is composed of \(L_{fi}\) and \(C_{fi}\). Note that the voltage measurements \(V_i\) and \(V_i^*\) are taken up and downstream of the coupling inductance \(L_i\), respectively, and these are used to estimate the angle phase (\(\theta_i\) and \(\theta_i^*\)) and frequency (\(\omega_i\) and \(\omega_i^*\)) over each node.

**Assumption 1.** In this thesis it is assumed a general framework, where the microgrid is always balanced and three-phased without a neutral wire. Moreover, the primary control level
is assumed in a dq framework, although it is not a necessary condition for the secondary level. The design concerns related with the primary controllers used on simulations and experimental tests, are included in the appendix A2.

Figure 3.1: DMPC for Frequency Regulation and Active Power Consensus - Control Scheme.

Considering that all interactions between the $DG_i$ and the microgrid are through $L_i$, it is possible to state the equations (3.1), (3.2) and (3.3).

$$\omega_i(t) = \omega_0 + M_{\omega i} P_i(t) + \omega_{s,i}(t)$$  \hspace{1cm} (3.1)

$$\delta\theta_i(t) = \theta_i(t) - \theta_i^*(t) = \int_0^t \left[\omega_i(\tau) - \omega_i^*(\tau)\right] d\tau$$  \hspace{1cm} (3.2)

$$P_i(t) = B_i V_i(t)V_i^*(t) \sin(\delta\theta_i(t))$$  \hspace{1cm} (3.3)

Equation (3.1) is the frequency droop control law, which changes the operating point, $\omega_i$, from the nominal value, $\omega_0$, in order to supply the active power demanded, $P_i$. The droop equation is included into the model because it establishes a linear relationship between $\omega_i$ and $P_i$, where $M_{\omega i}$ is the droop slope, and $\omega_{s,i}$ represents the secondary control action, used to restore the frequency to $\omega_0$. The angle phase deviation, $\delta\theta_i$, caused by $L_i$ is stated from equation (3.2). In this case, $\delta\theta_i$ is used to estimate the active power transferred from $DG_i$ to the microgrid. Finally, to achieve the power consensus in the microgrid, the power contribution from each $DG$, is defined by equation (3.3), where $B_i = 1/L_i\omega_0$.

**Assumption 2.** Equation (3.3) is stated from the classical power systems approach, to compute the power transferred from a generator to the electrical system. Under this context,
the line impedance between the generator and the reference node is assumed as strongly induc-
tive. Although the short line distances in the microgrids context produces a resistive-inductive
impedance, techniques such as virtual impedance introduced in [64, 65], are used in micro-
grids to compensate the resistive effects, allowing to use the assumption of an inductive line
[66, 67, 18].

Note that using equations (3.1), (3.2) and (3.3) as a base for the predictive controller, nei-
ther a model of the whole microgrid, nor a model of every type of generator in the microgrid,
such as it was used in [58, 59] and [60, 61] respectively, are required.

3.1.2 Discrete Time Model

Because the predictive control design requires a discrete time model to predict the DG behav-
ior, equations (3.1), (3.2) are discretized using forward Euler method, where
\[ t_n = nT_{sec}, n \in \mathbb{Z}^+, \]
and \[ T_{sec}. \]

Although non-linear constraints can be used in some optimization problems, the com-
putational effort required to solve this type of problems is higher, therefore, in this case
only linear constraints will be included in the QP problem, in order to preserve the so-
lution period according to the secondary control requirements. Regarding it, and before
 to be discretized, the equation (3.3) is linearized around the measured/estimated point \[ P_i = \{\omega_i(t_n), \omega_i^*(t_n), V_i(t_n), V_i^*(t_n), \theta_i(t_n), P_i(t_n), Q_i(t_n)\}. \] Then, the discretized model is de-

fined by equations (3.4), (3.5) and (3.6), whereas the discretization and linearization proce-
dures are detailed in the appendix A3.

\begin{align*}
\omega_i(t_{n+1}) &= \omega_i(t_n) + M_{p,i} [P_i(t_{n+1}) - P_i(t_n)] + \Delta \omega_{s,i}(t_n) \quad (3.4) \\
\delta \theta_i(t_{n+1}) &= \delta \theta_i(t_n) + T_{sec} [\omega_i(t_{n+1}) - \omega_i^*(t_n)] \quad (3.5) \\
P_i(t_{n+1}) &= P_i(t_n) + [\delta \theta_i(t_{n+1}) - \delta \theta_i(t_n)] B_i V_i(t_n) V_i^*(t_n) \cos(\delta \theta_i(t_n)) \quad (3.6)
\end{align*}

Though (3.1) and (3.3) are static equations, these should be discretized, as well as (3.2),
in order to state the prediction model; in this case the terms \[ \omega_i(t_{n+1}), \delta \theta_i(t_{n+1}), \] and \[ P_i(t_{n+1}), \]
from (3.4) and (3.6) and (3.5), can be considered as predicted values at one-step ahead. Note
that equation (3.4) includes the incremental operator \( \Delta \), defined by equation (3.7), and it is
related with the secondary control action \( \omega_{s,i} \). This operator is included in order to mitigate
the steady state error including an integral action at the controller output, as it is shown in
Fig. 3.1

\[ \Delta f(t_n) = [f(t_n) - f(t_{n-1})] \quad (3.7) \]

3.1.3 Communication Network Model

In order to include the communication features into the optimization problem, a communica-
tion model is built considering an adjacency matrix, and the delay caused by the networking
and automation protocols (end-end communication delay). The adjacency term, $a_{ij}$ and the adjacency matrix $A$, are defined by equations (3.8) and (3.9), respectively, where $p$ indicates the number of DGs in the microgrid.

At the rest of this thesis, $DG_i$ is also referred as the local unit, whereas $DG_j$ is referred as neighbor unit; then the neighborhood of $DG_i$ is defined as the set of DGs that established communication with $DG_i$, and is represented by the $i-th$ row of $A$. Note that, although the matrix $A$ is similar than the network representation reported in [3], definition (3.9) implies that each DG updates its neighborhood as a function of the received information at each sampling period. This last feature, not only allows to represent every communication topology, but it brings robustness against issues such as data dropouts and unavailability of communications links. The adjacency matrix properties are detailed in [39].

$$a_{ij}(t_n) = \begin{cases} 1 & \text{Data from } DG_j \text{ arrives to } DG_i \text{ at } t_n \\ 0 & \text{Data from } DG_j \text{ does not arrive to } DG_i \text{ at } t_n \\ 0 & j = i \end{cases} \quad \forall j \in \{1...p\}$$

$$A(t_n) = \begin{bmatrix} a_{11}(t_n) & a_{12}(t_n) & \cdots & a_{1p}(t_n) \\ a_{21}(t_n) & a_{22}(t_n) & \cdots & a_{2p}(t_n) \\ \vdots & \vdots & \ddots & \vdots \\ a_{p1}(t_n) & a_{p2}(t_n) & \cdots & a_{pp}(t_n) \end{bmatrix}$$

(3.8) (3.9)

The delay term ($\tau_{ij} \geq 1$) is quantized as an integer multiple of the sampling period $T_{sec}$, and represents the time required for the transmission-reception process between $DG_i$ and $DG_j$ (networking protocol), as well as, the time required by the automation protocol to process the data-frame. Note that this representation does not imply to use of specific communication protocols.

**Assumption 3.** It is assumed a symmetric and full duplex communication network. It implies that $\tau_{ij} = \tau_{ji}$, and the adjacency matrix $A$ is symmetric, then $a_{ij} = a_{ji}$ [39].

### 3.1.4 Optimization Problem for Predictive Control

Predictive control solves an optimization problem to minimize a cost function, considering a set of equalities and inequalities to bound the space where the optimal solution is searched. In this thesis, a quadratic cost function is used, and the set of equalities and inequalities constraints are built from the discretized models presented in section 3.1.2 and 3.1.3 and operational constraints. The optimal solution, $X_i$, contains the predicted control sequence, as well as the predicted trajectory for the $DG_i$, along the control and prediction horizons, $N_u$ and $N_y$, where $N_u \leq N_y$, respectively.

- **Equality constraints:** The set of equality constraints shown in (3.10), (3.11) and (3.12) is derived from equations (3.4), (3.5) and (3.6), respectively; and it is used to predict the $DG_i$ behavior at $t_{n+k}$, where $k \in \mathbb{Z}^+$. Note that the expression $B_iV_i(t_n)V_i^*(t_n)$
Cost function: 

$$\omega_i(t_{n+k}) = \omega_i(t_{n+k-1}) + M_{\omega,i} [P_i(t_{n+k}) - P_i(t_{n+k-1})] + \Delta \omega_{s,i}(t_{n+k-1})$$ (3.10)

$$\delta \theta_i(t_{n+k}) = \delta \theta_i(t_{n+k-1}) + T_{sec} [\omega_i(t_{n+k}) - \omega_i^*(t_n)]$$ (3.11)

$$P_i(t_{n+k}) = P_i(t_n) + [\delta \theta_i(t_{n+k}) - \delta \theta_i(t_n)] B_i V_i(t_n) V_i^*(t_n) \cos(\delta \theta_i(t_n))$$ (3.12)

- Operational constraints: The set of operational constraints includes equality and inequality constraints. Equality (3.13) is a local estimation of the average frequency in the microgrid. Note that the adjacency terms, $a_{ij}$, modify the amount of neighbors and their frequencies considered for the local estimation on $DG_i$; whereas the communication delay is compensated by the latency estimation $\tilde{\tau}_{ij}$. Equation (3.14) forces that, for feasible solutions, the frequency trajectory converges to the nominal frequency $\omega_0$ at the end of the horizon. Finally, the inequality (3.15) ensures that the active power transferred from $DG_i$ to the microgrid remains within the feasible range.

$$\overline{\omega}_i(t_{n+k}) = \frac{\omega_i(t_{n+k}) + \sum_{j=1}^p a_{ij}(t_n) \omega_j(t_{n+k-\tilde{\tau}_{ij}})}{1 + \sum_{j=1}^p a_{ij}(t_n)}$$ (3.13)

$$\overline{\omega}_i(t_{n+N_y}) = \omega_0$$ (3.14)

$$0 \leq P_i(t_{n+k}) \leq P_{i \text{max}}$$ (3.15)

- Cost function: The cost function (3.16) is built from three weighted terms, which represent the control objectives. The first term refers to the average frequency regulation in the microgrid; the second one penalizes the control action required to achieve the control objectives; whereas the third term pursues the consensus among the active power contribution from $DG_i$ and its neighboring DGs.

$$J_i(t_n) = \sum_{k=1}^{N_y} \lambda_{1i}(\overline{\omega}_i(t_{n+k}) - \omega_0)^2 + \sum_{k=1}^{N_u} \lambda_{2i}(\Delta \omega_{s,i}(t_{n+k-1}))^2$$

$$+ \sum_{j=1, j \neq i}^{p} \sum_{k=1}^{N_y} \lambda_{3ij} a_{ij}(t_n) \left( \frac{P_i(t_{n+k})}{P_{i \text{max}}} - \frac{P_j(t_{n+k-\tilde{\tau}_{ij}})}{P_{j \text{max}}} \right)^2$$ (3.16)

To tune the predictive controller, the relationship among the weighting factors $\lambda_{1i}, \lambda_{2i}, \lambda_{3ij}$ and the desired microgrid performance, should be stated. Considering that the case $\lambda_{1i} = \mu_i \lambda_{3ij}$ shows a first order response for both, frequency regulation and power consensus on $DG_i$, where $\mu_i$ depends on the microgrid features; intuitive guidelines can be defined to set the weighting factors. The proposed guidelines are listed bellow, and it is desirable to use these in combination with heuristic methods, such as particle swarm optimization (PSO) [68] or branch and bound [69]:
\(-\lambda_{1i} > \mu_i \lambda_{3i}\): This case implies that the DG prioritizes the frequency regulation over the active power consensus. This setup is suggested for DGs where a higher active power-frequency droop slope, \(M_{p\omega,i}\), produces significant frequency deviations for small power changes.

\(-\lambda_{1i} < \mu_i \lambda_{3i}\): It is the opposite of the previous case. With this configuration the DG prioritizes the active power consensus. This setup is suggested for DGs where a lower active power-frequency droop slope, \(M_{p\omega,i}\), produces small frequency changes for big changes on real power.

\(-\lambda_{2i} > \lambda_{2j}\): This configuration is settled when a slower response for \(DG_i\), compared with the \(DG_j\) response, is desired. To increase \(\lambda_{2i}\) implies that an aggressive control action of \(DG_i\) is expensive, therefore the frequency regulation and the active power consensus will require more time to be achieved.

\(-\lambda_{1i} = 0\) or \(\lambda_{3i} = 0\): This case implies that the \(DG_i\) only contributes to the power consensus or to the frequency regulation, respectively. It is a non-desired configuration because in a worst case communication scenario, where \(a_{ij} = 0\) \(\forall j \in \{1...p\}\), it allows significant changes for frequency or real power in order to achieve the settled control objective.

\(-\lambda_{2i} = 0\): It is a non-allowed configuration because it implies that hard changes on the control action \(\Delta \omega_{s,i}\) are allowed, producing significant oscillations, even undamped oscillations, in the DG response.

In subsection [3.2.1] simulations explain the effects of each case.

- **Quadratic programming (QP) formulation:** Considering the equality and operational constraints, and the cost function, a QP problem can be stated. It is shown in equation (3.17); where the output vector \(X_i\) is defined by (3.18), and contains the predicted trajectories for the system variables and the predicted control sequence; note that this vector is turned to \(X_{ij}\) once it passes through the communication network, as it is shown in Fig. 3.1. Equations (3.19) to (3.24) define the matrices \(H_i, F_i, A_i, B_i, A_{eq,i}\) and \(B_{eq,i}\), respectively, where \(\mathbb{I}_{N_y}\) represents an identity matrix with \(N_y \times N_y\) entries, and \(0_{a \times b}\), is a \(a \times b\) matrix, with all entries settled in zero; whereas the intermediate terms used, are defined by equations (3.25) to (3.29). The procedure to synthesize the QP problem from the cost function, and in/equality constraints is shown in the appendix [A4].

\[
\begin{align*}
\minimize_{\bar{X}_i} & \quad J_i(t_n) := \frac{1}{2} X_i^T H_i X_i + F_i^T X_i \\
\text{subject to} & \quad A_i X_i \leq B_i \\
& \quad A_{eq,i} X_i = B_{eq,i}
\end{align*}
\]

\[
X_i = \begin{bmatrix}
\bar{\omega}(t_{n+1}), ..., \bar{\omega}(t_{n+N_y}), \Delta \omega_{s,i}(t_n), ..., \Delta \omega_{s,i}(t_{n+N_u-1}), \omega_i(t_{n+1}), ..., \omega_i(t_{n+N_y}), \\
\delta \theta_i(t_{n+1}), ..., \delta \theta_i(t_{n+N_y}), P_i(t_{n+1}), ..., P_i(t_{n+N_y})
\end{bmatrix}^T
\]
\[
H_i(t_n) = \begin{bmatrix}
2\lambda_1 I_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_u \times N_y} & 2\lambda_2 I_{N_y} & 0_{N_u \times N_y} & 0_{N_y} & 0_{N_u \times N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & \frac{2\lambda_N P_i(t_n) p_{i \max}}{p_{i \max}^2 I_{N_y}}
\end{bmatrix}_{((4N_y + N_u) \times (4N_y + N_u))}
\]

\[
F_i(t_n) = \begin{bmatrix}
-2\lambda_1 \omega_0 I_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & \frac{2\lambda_N P_i(t_n) p_{i \max}}{p_{i \max}^2} C_{p,i}(t_n)
\end{bmatrix}^T_{(1 \times (4N_y + N_u))}
\]

\[
A_i(t_n) = \begin{bmatrix}
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & -I_{N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & I_{N_y}
\end{bmatrix}_{((2N_y \times (4N_y + N_u))}
\]

\[
B_i(t_n) = \begin{bmatrix}
0_{N_y \times 1} \\
P_{i \max} I_{N_y} \times 1
\end{bmatrix}_{(2N_y \times 1)}
\]

\[
A_{eq,i}(t_n) = \begin{bmatrix}
I_{N_y} & 0_{N_y \times N_u} & \frac{-1}{1 + \Gamma_i(t_n)} (I_{N_y} - 1 | 0)_{N_y \times N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_y} & -(I_{N_y} | 0)_{N_y \times N_u} & T_{N_y} & 0_{N_y} & -M_{p_{i \omega,i}} T_{N_y} \\
0_{N_y} & 0_{N_y \times N_u} & -T_{sec} I_{N_y} & T_{N_y} & 0_{N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & -K_{\delta \theta}(t_n) I_{N_y} & I_{N_y}
\end{bmatrix}
\]

\[
B_{eq,i}(t_n) = \begin{bmatrix}
\sum_{j=0}^{p} a_{ij}(t_n) \omega_j(t_n + 1 - \tau_{ij}) \\
\vdots \\
\sum_{j=0}^{p} a_{ij}(t_n) \omega_j(t_n + N_y - \tau_{ij}) \\
\sum_{j=0}^{p} a_{ij}(t_n) \omega_j(t_n) \\
\omega_i(t_n) - M_{p_{i \omega,i}} P_i(t_n) \\
0 \\
\vdots \\
0 \\
\delta \theta_i(t_n) - T_{sec} \omega^*_i(t_n) \\
- T_{sec} \omega^*_i(t_n) \\
\vdots \\
- T_{sec} \omega^*_i(t_n) \\
P_i(t_n) - K_{\delta \theta}(t_n) \delta \theta_i(t_n) \\
\vdots \\
P_i(t_n) - K_{\delta \theta}(t_n) \delta \theta_i(t_n)
\end{bmatrix}
\]

\[
\Gamma_i(t_n) = \sum_{j=1, j \neq i}^{p} a_{ij}(t_n)
\]
\[ C_{pi}(t_n) = \left[ \sum_{j=1,j \neq i}^{p} \frac{a_{ij}(t_n)P_j(t_{n+1-\tau_{ij}})}{P_{j \text{ max}}} \ldots - \sum_{j=1,j \neq i}^{p} \frac{a_{ij}(t_n)P_j(t_{n+N_{y-\tau_{ij}}})}{P_{j \text{ max}}} \right] \] (3.26)

\[ (\mathbb{I}_{N_u}|0)_{N_u \times N_u} = \begin{bmatrix} \mathbb{I}_{N_u} \\ 0_{(N_{y-N_u}) \times N_u} \end{bmatrix}_{(N_y \times N_u)} \] (3.27)

\[ T_{Ny} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{(N_y \times N_y)} \] (3.28)

\[ K_{\delta \theta}(t_n) = B_iV_i(t_n)V_i^*(t_n) \cos(\delta \theta_i(t_n)) \] (3.29)

Note that the submatrix \((\mathbb{I}_{N_u}|0)\), shown in (3.23), associated with the predictions of the droop equation, implies that predictions \(\Delta \omega_{s,i}(t_{n+N_u})\) to \(\Delta \omega_{s,i}(t_{n+N_y})\) are zero; and the matrix \(T_{Ny}\), defined in (3.28) as a Toeplitz matrix, is used to state the difference between predicted variables at \(t_{n+k}\) and \(t_{n+k-1}\).

As mentioned in [24], a feasible solution to the optimization problem is required for a stable predictive control. Note that the constraints (3.14) and (3.15), for terminal frequency and power range, respectively, are related to the QP feasibility, ensuring that the \(DG_i\) does not overpass its physical limits along the whole prediction horizon. To ensure a feasible initial condition, the DMPC is enabled when the microgrid operates close to the nominal frequency \(\omega_0\); however, if a non-feasible solution is obtained to the QP problem, \(\Delta \omega_{s,i}(t_{n+1})\) is settled to zero in order to do not change the controller output. In a black start scenario, the state required to enable the DMPC is achieved when the primary control level operates without load.

The power range and terminal frequency constraints limit the feasible solution space of the QP problem; then, a reduction on the computational cost is achieved when the problem is solved [24]. To compute the QP solution, the QPKWIK algorithm, which is based on the classic active-set method, is used [70]. A detailed explanation of how the QP problem is updated and solved is shown in Algorithm 1.

3.2 Simulation Results

In this section, two sets of simulations are presented. The first set refers to the tuning hints, for the proposed DMPC, introduced in 3.1.4 whereas the second set of simulations presents a comparison between the proposed DMPC and the DAPI controller in three common scenarios. The simulation environment were developed using the PLECS Blockset® and MATLAB Simulink® platforms, for electrical and control layers, respectively. The simulator is closest to a real microgrid, so modeling details such as the inner loops, droop controllers, LC filters, resistive loads, and switch snubbers are implemented on this.
Algorithm 1 DMPC solution for each $DG_i$

Inputs:
- Measurements and estimations: $\{\omega_i(t_n), \omega^*_i(t_n), V_i(t_n), V^*_i(t_n), \delta\theta_i(t_n), P_i(t_n)\}$
- Received information: $X_{ij}$, $j = \{1, \ldots, p\}$

Outputs: $X_i, \Delta\omega_s,i(t_n)$

Initialization:
1. Compute matrix coefficients of $H_i, F_i, A_i, A_{eq,i}, B_i, B_{eq,i}$
2. for every $t_n$ do
3. Compute adjacency terms $a_{ij}$ according to the received information.
4. According to the received information, compute the sums of frequency, and real power stated by (3.24) and (3.26), respectively.
5. Update matrices $H_i, F_i, A_i, A_{eq,i}, B_i, B_{eq,i}$ from (3.17) according to the results of step 4 and the measurements/estimations $\{\omega_i(t_n), \omega^*_i(t_n), V_i(t_n), V^*_i(t_n), \delta\theta_i(t_n), P_i(t_n)\}$.
7. if $X_i$ is feasible and $t < t_n + T_{sec}$ then
8. Extract $\Delta\omega_{s,i}(t_n)$ from $X_i$.
9. else $\Delta\omega_{s,i}(t_n) = 0$
10. end if
11. Update controller outputs and send $X_i$ to neighbor DGs if it is feasible
12. end for

3.2.1 Tuning Effects on the Microgrid Performance

To illustrate the effects of the weighting factors, $\lambda_{1i}, \lambda_{2i}$ and $\lambda_{3i}$, on the system performance, a resistive load impact is used as disturbance over a microgrid with two DGs. The microgrid diagram is shown in Fig. 3.2, whereas the electrical and control parameters are shown in Table 3.1 and Table 3.2.

![Figure 3.2: Two DGs Microgrid - Tuning Effects on the Microgrid Performance.](image)

Table 3.1: Microgrid Electrical Parameters - Microgrid Two DGs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{prim}$</td>
<td>Primary Level Sampling Period</td>
<td>1/16E3 s</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>Load 1</td>
<td>5.25 $\Omega$</td>
</tr>
<tr>
<td>$L_i$</td>
<td>Coupling Inductance</td>
<td>2.5 mH</td>
</tr>
<tr>
<td>$L_{fi}$</td>
<td>Filter Inductance</td>
<td>0.85 mH</td>
</tr>
<tr>
<td>$C_{fi}$</td>
<td>Filter Capacitance</td>
<td>70 $\mu$F</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Nominal Frequency</td>
<td>314.159 rad/s</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Nominal Voltage (peak)</td>
<td>150 V</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Cut-off Frequency - Droop Controller</td>
<td>2$\pi$ rad/s</td>
</tr>
</tbody>
</table>
Table 3.2: Power Capacities and Droop Slopes - Microgrid Two DGs

<table>
<thead>
<tr>
<th>Power Capacities and Droop Slopes</th>
<th>DG1</th>
<th>DG2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{max}}$ [KW] Power Capacity</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td>$M_{p\omega}$ [$\frac{\text{rad}}{\text{s}}$] P-$\omega$ Droop Slope</td>
<td>-1E-4</td>
<td>-0.85E-4</td>
</tr>
<tr>
<td>$M_{q\nu}$ [$\frac{\text{VAR}}{\text{V}}$] Q-V Droop Slope</td>
<td>-1.2E-3</td>
<td>-1.2E-3</td>
</tr>
</tbody>
</table>

- $\lambda_{1i} = \mu_i \lambda_{3i}$, $\lambda_{1i} > \mu_i \lambda_{3i}$, and $\lambda_{1i} < \mu_i \lambda_{3i}$: In Fig. 3.3, the $DG_1$ performance, using DMPC, is compared when different values of $\lambda_{11}$ and $\lambda_{31}$ are used, while the $DG_2$ parameters are preserved. A first order response is achieved for frequency regulation and active power consensus when $\lambda_{11} = \mu_1 \lambda_{31}$ ($\lambda_{11} = 1E - 1 \left[ \frac{s}{\text{rad}} \right]^2$, $\lambda_{31} = 1E - 2$, $\mu_1 = 10 \left[ \frac{s}{\text{rad}} \right]^2$); it is assumed as the comparison base. When $\lambda_{11}$ is increased ($\times 3$), the frequency regulation is prioritized, therefore a second order response is attained; however, because the frequency should be unique in the microgrid, power oscillations are generated, causing a more aggressive response in the power consensus as well. In the other hand, when $\lambda_{11}$ is lower ($\times 5$) than $\mu_1 \lambda_{31}$, a faster response on the power consensus is achieved, however, in this case, it does not represent significant changes on the frequency regulation.

![Microgrid Frequency - DMPC Control - Weighting Factors Comparison](image1)

![Microgrid Active Power - DMPC Control - Weighting Factors Comparison](image2)

Figure 3.3: $DG_1$ Response With Different Values of $\lambda_{11}$ and $\lambda_{31}$.

- $\lambda_{2i} > \lambda_{2j}$: In this comparison, $\lambda_{22}$ is greater than $\lambda_{21}$ ($\lambda_{21} = 1 \left[ \frac{s}{\text{rad}} \right]^2$, $\lambda_{22} = 3 \left[ \frac{s}{\text{rad}} \right]^2$), implying an expensive cost for changes on the control action of $DG_2$; therefore the microgrid response on both control objectives, frequency regulation and power consensus, turns slower than in the base case. Because to achieve the power consensus is expensive on $DG_2$, $DG_1$ compensates it in order to supply the power demanded by the load, causing an overshoot in its power contribution, as is shown in Fig. 3.4.
Figure 3.4: $DG_1$ Response With $\lambda_{22} > \lambda_{21}$.

- $\lambda_{1i} = 0$: In this case only the power consensus is enabled, therefore, the steady state frequency is established by the droop control. As is shown in Fig. 3.5, the active power performance preserves the first order response, however, when the communication between $DG_1$ and $DG_2$ fails, the power consensus cannot be achieved. In this latter case, the DMPC only minimizes the control action changes.

Figure 3.5: Microgrid Response With $\lambda_{11} = \lambda_{12} = 0$.

- $\lambda_{3i} = 0$: In Fig. 3.6 the effects when the power consensus is disabled, are shown. In this
case frequency regulation is achieved with and without communication between $DG_1$ and $DG_2$ because, according to equation (3.13), the local estimation of the average frequency is equal to the local measurement, $\bar{\omega}_i = \omega_i$, in absence of communication, then the regulation turns into a local task. Although in both cases is achieved the frequency regulation, when the DGs cooperate between them, the frequency and power responses are less oscillatory than the no-communication scenario.

$$K_{i,\omega} = K_{i,\omega}(t_{n+1}) - T_{sec}(\omega_i(t_n) - \omega_0) - T_{sec} \sum_{j=1,j \neq i}^p a_{ij}(t_n) \left( \frac{P_i(t_{n+k})}{P_{i,max}} - \frac{P_j(t_{n+k-\tau_{ij}})}{P_{j,max}} \right)$$  \hspace{1cm} (3.30)
Figure 3.7: Four DGs Microgrid - Performance Test.

Table 3.3: Microgrid Electrical Parameters - Microgrid Four DGs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{prim}$</td>
<td>Primary Level Sampling Period</td>
<td>$1/16E3$ s</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>Load 1 - Test 3.1</td>
<td>$5.25 , \Omega$</td>
</tr>
<tr>
<td></td>
<td>Load 1 - Test 3.2 and Test 3.3</td>
<td>$10.5 , \Omega$</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>Load 3 - Test 3.1</td>
<td>$11 , \Omega$</td>
</tr>
<tr>
<td></td>
<td>Load 3 - Test 3.2 and Test 3.3</td>
<td>$22 , \Omega$</td>
</tr>
<tr>
<td>$L_i$</td>
<td>Coupling Inductance</td>
<td>$2.5 , mH$</td>
</tr>
<tr>
<td>$L_{ij}$</td>
<td>Transmission Line Inductance</td>
<td>$2.5 , mH$</td>
</tr>
<tr>
<td>$L_{f_1}$</td>
<td>Filter Inductance</td>
<td>$0.85 , mH$</td>
</tr>
<tr>
<td>$C_{f_1}$</td>
<td>Filter Capacitance</td>
<td>$70 , \mu F$</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Nominal Frequency</td>
<td>$314.159 , rad/s$</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Nominal Voltage (peak)</td>
<td>$150 , V$</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Cutoff Frequency - Droop Controller</td>
<td>$1.256 , rad/s$</td>
</tr>
</tbody>
</table>

Table 3.4: Power Capacities and Droop Slopes - Microgrid Four DGs

<table>
<thead>
<tr>
<th>Power Capacities and Droop Slopes</th>
<th>DG1</th>
<th>DG2</th>
<th>DG3</th>
<th>DG4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{max}$ [KW] Power Capacity</td>
<td>4.0</td>
<td>3.5</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td>$M_p\omega$ [rad W] P-$\omega$ Droop Slope</td>
<td>-1E-4</td>
<td>-0.85E-4</td>
<td>-0.85E-4</td>
<td>-1E-4</td>
</tr>
<tr>
<td>$M_qV$ [V VAR] Q-V Droop Slope</td>
<td>-1.2E-2</td>
<td>-1.2E-2</td>
<td>-1.2E-2</td>
<td>-1.2E-2</td>
</tr>
</tbody>
</table>

Table 3.5: DMPC General Parameters - Microgrid Four DGs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{sec}$</td>
<td>Secondary Level Sampling Period</td>
<td>0.1 s</td>
</tr>
<tr>
<td>$\hat{\tau}_{ij}$</td>
<td>Estimated Communication Delay</td>
<td>0.1 s</td>
</tr>
<tr>
<td>$N_y$</td>
<td>Prediction Horizon</td>
<td>10</td>
</tr>
<tr>
<td>$N_u$</td>
<td>Control Horizon</td>
<td>10</td>
</tr>
</tbody>
</table>

The tuning process were performed using the guidelines of section 3.2.1 for DMPC, and [3] for DAPI controllers. In both cases PSO algorithm, considering the fitness function for frequency regulation (3.31), was used to the fine tuning. The microgrid is disturbed, with two
load impacts, according with the test 3.1 explained bellow. The first load impact is applied at \( t = t_{\text{dis}} \) and it is considered as the initial point in the fitness function. The tuned parameters for DMPC and DAPI schemes are presented in Table 3.6 and Table 3.7 respectively.

\[
e_{fit} = \sum_{t=t_{\text{dis}}}^{T_{\text{sim}}} t \left[ \omega(t) - \omega_0 \right]^2
\]

(3.31)

Table 3.6: DMPC Weighting Factors - Microgrid Four DGs

<table>
<thead>
<tr>
<th>Weighting Factors</th>
<th>DG1</th>
<th>DG2</th>
<th>DG3</th>
<th>DG4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_1 \left[ \frac{\text{ms}}{\text{rad}} \right]^2 ) Average Frequency Error</td>
<td>3E0</td>
<td>3E0</td>
<td>3E0</td>
<td>3E0</td>
</tr>
<tr>
<td>( \lambda_2 \left[ \frac{\text{ms}}{\text{rad}} \right]^2 ) Frequency Control Action</td>
<td>1.1E0</td>
<td>1.9E0</td>
<td>1.5E0</td>
<td>1.2E0</td>
</tr>
<tr>
<td>( \lambda_3 ) Active Power Consensus</td>
<td>9E-4</td>
<td>1.2E-2</td>
<td>3E-3</td>
<td>3E-3</td>
</tr>
</tbody>
</table>

Table 3.7: DAPI Parameters - Microgrid Four DGs

<table>
<thead>
<tr>
<th>Weighting Factors</th>
<th>DG1</th>
<th>DG2</th>
<th>DG3</th>
<th>DG4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{i,\omega} \ [s] ) DAPI Integral Gain</td>
<td>2.7</td>
<td>2.6</td>
<td>2</td>
<td>3.5</td>
</tr>
</tbody>
</table>

- **Test 3.1: Resistive load impacts.** As it was mentioned, this test was used by the PSO algorithm to tune the microgrid controllers. The first load impact connects the resistive loads \( Z_1 \) and \( Z_3 \) simultaneously, rising the total active power demanded from 0% to 90% at \( t = 15 \text{s} \), whereas the second disturbance disconnects \( Z_1 \), at \( t = 25 \text{s} \), reducing to 60% the power demanded. The average frequency response, using DMPC and DAPI schemes is presented in Fig. 3.8 whereas Fig. 3.9 presents the active power response, where each power value is normalized according with the maximum DG capacity.

From the results of this scenario is stated that both controllers have similar settling time for power consensus, whereas DAPI controller has a faster response on frequency regulation; however, it presents oscillations when the load decreases. This behavior is related to the frequency error term, whereas, for each DMPC controller the frequency predictions, from the neighboring controllers are included, in the DAPI controllers just is considered the local measurement, implying more aggressive actions. In the same way, poor communication, also implies aggressive actions, as it is noted in the Fig. 3.9 for the \( DG_2 \) response, using the DAPI scheme. In this case, \( DG_2 \) has communication only with \( DG_1 \), implying a poor knowledge about the global microgrid behavior, then the \( DG_2 \) tries to supply the power demanded, immediately the microgrid is disturbed. Notice in Fig. 3.9 that, although the power consensus is achieved latter, when DAPI is used, the \( DG_2 \) power overshot overpass its total capacity, whereas for the DMPC scheme all DGs satisfy the active power constraint defined in (3.15).
Figure 3.8: Average Frequency Responses Using DMPC and DAPI Schemes.

Figure 3.9: Active Power Responses Using DMPC and DAPI Schemes.

- **Test 3.2:** Communication network turned off. In this scenario, the microgrid is operating at 45% of its capacity (loads $Z_1$ and $Z_3$ are enabled), and suddenly the whole
communications network is turned off at \( t = 25 \) s, for 1.5 seconds. At \( t = 26 \) s the total load decreases to 30% (load \( Z_1 \) is disabled). This scenario is the worst case in terms of data losses or communication topology changes and, it implies a decentralized and non-cooperative operation while a load impact occurs. As the communications network is turned off, all the entries \( a_{ij} \) of the adjacency matrix are turned null. Then, the power consensus is not working while the failure happens and, the frequency regulation for both schemes is based on only the local frequency measure as well.

Fig. 3.10 shows that when the communication fails, both control schemes regulate the microgrid frequency, even after the load impact. Both control schemes present an overshoot caused by the load decreasing and, also both schemes have a similar response for frequency regulation when the communication is restored. However, the DAPI response in Fig. 3.10 is less oscillatory than the response in Fig. 3.8, whereas the DMPC response is similar in both cases. This implies that the DAPI scheme is more sensitive than the DMPC when the load impacts are strong.

Fig. 3.11 shows the power-sharing response, which is similar in this case for both schemes while the communication failure is enabled. It is because both schemes regulate the local frequency while the failure happens, and this does not imply significant changes in power response. When the droop transient is happening, the fault is cleared and, both schemes achieve the power consensus again, however, the DAPI response is little more oscillatory than the DMPC response, especially for the \( DG_2 \).

![Figure 3.10: Average Frequency Responses Using DMPC and DAPI Schemes When Communication Network Fails.](image-url)
• **Test 3.3: Changes in communication delay.** In this test, the same two load impacts used in the previous scenario, are applied. In this case the delay estimation, $\hat{\tau}_{ij}$, used in the DMPC controllers is preserved at 0.2s (DAPI controllers do not need a delay estimation), and the real delay, $\tau_{ij}$, changes between 0.1s and 1s, and considered as constant for each simulation.

Fig. 3.12 and Fig. 3.13 present the results for frequency regulation, whereas Fig. 3.14 and Fig. 3.15 show the active power response. Note that when the delay increases, the DMPC frequency response presents damped oscillations, within a $\pm 0.3\%$ range at the worst case, whereas the DAPI response is preserved. It is because DMPC controller uses information from neighboring DGs to compute the local average frequency estimation $\hat{\omega}_i$, instead to use just local measurement as DAPI controller.

About the power consensus, in both cases it is achieved, however, the DAPI response is more oscillatory because the delay affects the information sharing so the control actions try to achieve the power consensus based on past information. In contrast, as the future information is shared among neighboring DGs in the DMPC scheme, and this information is used to solve the local optimization problem, the communication delay effect is compensated in a better way than the DAPI scheme.
Figure 3.12: Microgrid Frequency Response Using DMPC Scheme With $\tau_{ij} = 0.2s$ (Top), $\tau_{ij} = 0.4s$ (Middle), $\tau_{ij} = 0.7s$ (Bottom).

Figure 3.13: Microgrid Frequency Response Using DAPI Scheme With $\tau_{ij} = 0.2s$ (Top), $\tau_{ij} = 0.4s$ (Middle), $\tau_{ij} = 0.7s$ (Bottom).
Figure 3.14: Microgrid Active Power Response Using DMPC Scheme With $\tau_{ij} = 0.2s$ (Top), $\tau_{ij} = 0.4s$ (Middle), $\tau_{ij} = 0.7s$ (Bottom).

Figure 3.15: Microgrid Active Power Response Using DAPI Scheme With $\tau_{ij} = 0.2s$ (Top), $\tau_{ij} = 0.4s$ (Middle), $\tau_{ij} = 0.7s$ (Bottom).
3.3 Discussion

In this chapter a non-iterative DMPC for frequency regulation and active power consensus was proposed, considering in its model the droop and power transfer equations, as well as a communication network model, to compensate for electrical and communication issues.

The optimization problem was stated as a canonical QP problem, including equality constraints based on the electrical and the communication model, and adding operational constraints, such as terminal frequency and a local average frequency estimation. Inequality constraints are included as well, to preserve the power contribution of each DG within its operational range.

Considering the microgrid behavior, a set of guidelines to tune the weighting factors of the cost function, were stated. This tuning process considers that a first order response is achievable when $\lambda_{1i} = \mu_i \lambda_{3i}$, where the value of $\mu_i$ depends on the microgrid features; and the values for $\lambda_{1i}$ and $\lambda_{3i}$ are adjusted according with the desired frequency and power consensus response, respectively.

The microgrid performance using the proposed DMPC was validated using simulation tests, and comparing the results with the performance obtained using a DAPI control scheme. Two remarkable differences between DMPC and DAPI schemes, were stated from the results: (i) the control effort is better distributed by the proposed DMPC, when strong load impacts disturb the microgrid; preserving the operation within operational limits of active power; and (ii) the proposed DMPC presents a better response for power consensus when the latency in the communication network increases; in this case, frequency regulation, using DAPI controller, is not affected because it is achieved using only local information. An additional remarkable topic from the results is that both DMPC, and DAPI schemes, present a similar response when the whole communication network fails. In this scenario, both controllers regulate the frequency using only local measurements, whereas the active power consensus is not enabled.
Chapter 4

DMPC for Frequency and Voltage Regulation

Distributed control schemes have transformed frequency and voltage regulation into a local task in distributed generators (DGs) rather than by a central secondary controller, although these variables, as well as the active and reactive power are co-dependent in the microgrid, the classical approach in secondary control considers these as decoupled variables. This chapter presents a distributed predictive control applied to the secondary level of microgrids, where the model used for prediction purposes is based on the co-dependent set of droop, and active and reactive power transfer equations; as well as the communication features, such as connectivity and latency, are also included, thus making the controller tolerant to electrical and communication failures. Frequency and voltage regulation, as well as the consensus over the active and reactive power contributions are considered as control objectives for each power unit in the microgrid, enlarging the scope proposed on chapter 3.

The experimental and simulation results show that the proposed scheme (i) responds properly to load variations, working within operating constraints, such as generation capacity and voltage range; (ii) maintains the control objectives when a power unit is disconnected and reconnected without any user updating in the controllers; (iii) compensates for the effects of communication issues over the microgrid dynamics; (iv) is scalable, allowing to integrate new DGs to the microgrid without mayor concerns; and, (v) presents a better performance than a decoupled DAPI controllers, achieving reactive power consensus and stable response when large latency affects the information sharing.

4.1 Model Used for Controller Design

4.1.1 Distributed Generator Model

Unlike previously reported DMPC for the secondary level of microgrids, the proposed model allows frequency and voltage regulation to be merged in the design of one multi-input multi-output distributed controller, and power consensus is achieved as well. This model is based on equations (3.1), (3.2), and (3.3), used for DMPC for frequency regulation, and equations
(4.1), and (4.2), included to predict the voltage, and reactive power trajectories, respectively. Moreover, the communication model, that represents connectivity and latency, presented in 3.1.3 is also included. The proposed DMPC also includes explicit operational constraints such as voltage range and apparent power limits.

\[ V_i(t) = V_0 + M_{qv,i}Q_i(t) + V_{s,i}(t) \]  

(4.1)

\[ Q_i(t) = B_i[V_i(t)^2 - V_i(t)V_i^*(t) \cos(\delta \theta_i(t))] \]  

(4.2)

Equation (4.1) represents the droop equation used to deviate the operating voltage, \( V_i \), from its nominal value, \( V_0 \), when the reactive power, \( Q_i \), demanded by the microgrid, changes. This equation determines the joint point between the primary and secondary control levels, because the secondary control action, \( V_{s,i} \), is stated to compensate the voltage deviation and to ensure the desired reactive power contribution from \( DG_i \).

To compute the reactive power contribution of \( DG_i \), is used the equation (4.2), which is stated considering the assumption 2 introduced in section 3.1.1. As like as the active power equation (3.3), equation (4.2) should be linearized and discretized, using Taylor expansion and forward Euler method, respectively, in order to be included as equality constraint in the QP problem.

### 4.1.2 Discrete Time Model

The complete discrete time model used to state the prediction model is shown in equations (4.3) to (4.7). Note that unlike equation (3.6), equation (4.6) includes a linear term respect to the voltage changes. It is possible to include this term, because equation (4.4) allows to state voltage predictions around the measured point \( \mathbb{P}_i \), defined in section 3.1.2. The detailed procedure to state the discretized model is presented in the appendix A3.

\[ \omega_i(t_{n+1}) = \omega_i(t_n) + M_{p\omega,i} [P_i(t_{n+1}) - P_i(t_n)] + \Delta \omega_{s,i}(t_n) \]  

(4.3)

\[ V_i(t_{n+1}) = V_i(t_n) + M_{qv,i} [Q_i(t_{n+1}) - Q_i(t_n)] + \Delta V_{s,i}(t_n) \]  

(4.4)

\[ \delta \theta_i(t_{n+1}) = \delta \theta_i(t_n) + T_{sec} [\omega_i(t_{n+1}) - \omega_i^*(t_n)] \]  

(4.5)

\[ P_i(t_{n+1}) = P_i(t_n) + [V_i(t_{n+1}) - V_i(t_n)] B_i V_i^*(t_n) \sin(\delta \theta_i(t_n)) \]  

\[ + [\delta \theta_i(t_{n+1}) - \delta \theta_i(t_n)] B_i V_i(t_n) V_i^*(t_n) \cos(\delta \theta_i(t_n)) \]  

(4.6)

\[ Q_i(t_{n+1}) = Q_i(t_n) + [V_i(t_{n+1}) - V_i(t_n)] B_i [2V_i(t_n) - V_i^*(t_n) \cos(\delta \theta_i(t_n))] \]  

\[ + [\delta \theta_i(t_{n+1}) - \delta \theta_i(t_n)] B_i V_i(t_n) V_i^*(t_n) \sin(\delta \theta_i(t_n)) \]  

(4.7)
4.1.3 Optimization Problem for Predictive Control

Using the QP structure presented in [3.1.4], the output vector, \( \mathbf{x}_i \), is redefined as is shown in equation (4.8). Note that this redefinition is mandatory to include the voltage and reactive power predictions as optimization variables, implying to extend the matrices \( H_i, F_i, A_i, B_i, A_{eq,i} \) and \( B_{eq,i} \) according with the prediction model stated from equations (4.9) to (4.17), and using the same procedure presented in the appendix [A4] for the frequency regulation case.

\[
\begin{align*}
\mathbf{x}_i &= \begin{bmatrix}
\bar{\omega}_i(t_{n+1}), \ldots, \bar{\omega}_i(t_{n+N_y}), V_i(t_{n+1}), \ldots, V_i(t_{n+N_y}), \\
\Delta \omega_{s,i}(t_n), \ldots, \Delta \omega_{s,i}(t_{n+N_{a,i}-1}), \omega_i(t_{n+1}), \ldots, \omega_i(t_{n+N_y}), \delta \theta_i(t_{n+1}), \ldots, \delta \theta_i(t_{n+N_y}), P_i(t_{n+1}), \ldots, P_i(t_{n+N_y}), \\
\Delta V_{s,i}(t_n), \ldots, \Delta V_{s,i}(t_{n+N_{a,i}-1}), V_i(t_{n+1}), \ldots, V_i(t_{n+N_y}), Q_i(t_{n+1}), \ldots, Q_i(t_{n+N_y})
\end{bmatrix}^T
\end{align*}
\]  
(4.8)

\[
\bar{\omega}_i(t_{n+k}) = \frac{\omega_i(t_{n+k}) + \sum_{j=1}^{p} a_{ij}(t_n) \omega_j(t_{n+k-\tau_j})}{1 + \sum_{j=1}^{p} a_{ij}(t_n)}
\]
(4.9)

\[
\bar{\omega}_i(t_{n+N_y}) = \omega_0
\]
(4.10)

\[
V_i(t_{n+k}) = \frac{V_i(t_{n+k}) + \sum_{j=1}^{p} a_{ij}(t_n) V_j(t_{n+k-\tau_j})}{1 + \sum_{j=1}^{p} a_{ij}(t_n)}
\]
(4.11)

\[
V_i(t_{n+N_y}) = V_0
\]
(4.12)

\[
\omega_i(t_{n+k}) = \omega_i(t_{n+k-1}) + M_{p\omega,i} [P_i(t_{n+k}) - P_i(t_{n+k-1})] + \Delta \omega_{s,i}(t_{n+k-1})
\]
(4.13)

\[
\delta \theta_i(t_{n+k}) = \delta \theta_i(t_{n+k-1}) + T_{sec} [\bar{\omega}_i(t_{n+k}) - \omega_i^*(t_n)]
\]
(4.14)

\[
P_i(t_{n+k}) = P_i(t_n) \\
+ [V_i(t_{n+k}) - V_i(t_n)] B_i V_i^*(t_n) \sin(\delta \theta_i(t_n)) \\
+ [\delta \theta_i(t_{n+k}) - \delta \theta_i(t_n)] B_i V_i(t_n) V_i^*(t_n) \cos(\delta \theta_i(t_n))
\]
(4.15)

\[
V_i(t_{n+k}) = V_i(t_{n+k-1}) + M_{q\omega,i} [Q_i(t_{n+k}) - Q_i(t_{n+k-1})] + \Delta V_{s,i}(t_{n+k-1})
\]
(4.16)

\[
Q_i(t_{n+k}) = Q_i(t_n) \\
+ [V_i(t_{n+k}) - V_i(t_n)] B_i [2V_i(t_n) - V_i^*(t_n) \cos(\delta \theta_i(t_n))] \\
+ [\delta \theta_i(t_{n+k}) - \delta \theta_i(t_n)] B_i V_i(t_n) V_i^*(t_n) \sin(\delta \theta_i(t_n))
\]
(4.17)

Equations (4.11), and (4.12), are included as operational constraints to predict the average voltage trajectory, and to state that it converges to the nominal value at the end of the
prediction horizon, \( N_y \). Two additional operational constraints are stated; inequality (4.18) is included to ensure that the predicted voltage trajectory is within its feasible range, whereas inequality (4.19) ensures that the predicted active and reactive power do not overpass the apparent power capacity, \( S_{\text{max}} \), of \( \text{DG}_i \). This last constraint is defined as a polytopic inner approximation of (4.20) using the triangular inequality; the detailed procedure to state (4.19) is shown in appendix A3.

\[
V_{\text{min}} \leq V_i(t_{n+k}) \leq V_{\text{max}}
\] (4.18)

\[
|P_i(t_n)| + |Q_i(t_n)| + \text{sign}(P_i(t_n)) (P_i(t_{n+k}) - P_i(t_n)) + \text{sign}(Q_i(t_n)) (Q_i(t_{n+k}) - Q_i(t_n)) \leq S_{\text{max}}
\] (4.19)

\[
|S_i(t)| = (P_i(t)^2 + Q_i(t)^2)^{1/2} < S_{\text{max}}
\] (4.20)

Regarding the cost function (3.16), equation (4.21) is stated, considering additional terms to regulate the average voltage, and to achieve the reactive power consensus simultaneously. These two control objectives are leaded by the control action \( \Delta V_{s,i} \), which is weighted by \( \lambda_{4i} \), and can be tuned conform to the guidelines for the frequency action \( \Delta \omega_{s,i} \), presented in section 3.1.4. Besides, the trade-off between the voltage regulation and the reactive power consensus, weighted by \( \lambda_{2i} \) and \( \lambda_{6i} \), also follows the relationships stated to tune the frequency and active power trade-off.

Note that the incremental operator is used in both, the cost function (4.21), and droop equation (4.16), implying to use an integrator at the controller output to mitigate the steady state error. Fig. 4.1 presents the control diagram used to implement the proposed DMPC. Consider that, although this controller uses an extended model, it does not imply more additional hardware than the measurement devices placed downstream to the coupling inductance \( L_i \) (adjacent measurement node), which are also used in the DMPC for only frequency regulation and real power consensus introduced in chapter 3.

\[
J_i(t_n) = \sum_{k=1}^{N_y} \left[ \lambda_{1i}(\bar{\omega}_i(t_{n+k}) - \omega_0)^2 + \lambda_{2i}(V_i(t_{n+k}) - V_0)^2 \right] + \sum_{k=1}^{N_u} \left[ \lambda_{3i}(\Delta \omega_{s,i}(t_{n+k-1}))^2 + \lambda_{4i}(\Delta V_{s,i}(t_{n+k-1}))^2 \right] + \sum_{j=1, j \neq i}^{p} \sum_{k=1}^{N_y} \lambda_{5ij}(t_n) \left( \frac{P_i(t_{n+k})}{|S_{i}\text{max}|} - \frac{P_j(t_{n+k-\tau_{ij}})}{|S_{j}\text{max}|} \right)^2 + \sum_{j=1, j \neq i}^{p} \sum_{k=1}^{N_y} \lambda_{6ij}(t_n) \left( \frac{Q_i(t_{n+k})}{|S_{i}\text{max}|} - \frac{Q_j(t_{n+k-\tau_{ij}})}{|S_{j}\text{max}|} \right)^2
\] (4.21)
4.2 Experimental setup and results

In this section experimental tests are presented in order to validate the controller performance in scenarios where: (i) load impacts are applied; (ii) a communication path fails; (iii) a DG is disconnected from and reconnected to the microgrid; and (iv) the latency (delay) in the communication network increases.

The experimental setup was built in the Microgrids Control Lab of the University of Chile. The setup uses PM15F120 and PM5F60 Triphase® modules to emulate a three-DG microgrid, and it is programmed using a Matlab-Simulink® interface. Each module is controlled by a real-time target (RTT) computer, where the DMPC for each DG is downloaded. External measurement devices were placed on the adjacent measurement nodes, and communicated with their respective RTTs using optical fiber and EtherCAT interfaces. A diagram of the setup is shown in Fig. 4.2 and a photo register is shown in Fig. 4.3 additional details about Triphase® modules are included in appendix A5. In Table 4.1 and Table 4.2, electrical and droop parameters are presented.

![Diagram](image)

**Figure 4.1: Control Scheme of DMPC for Frequency and Voltage Regulation.**

### Table 4.1: Microgrid Electrical Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{prim}$</td>
<td>Primary Level Sampling Period</td>
<td>1/16E3 s</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>Load 1</td>
<td>11 Ω</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>Load 2</td>
<td>22 Ω</td>
</tr>
<tr>
<td>$L_c$</td>
<td>Coupling Inductance</td>
<td>2.5 mH</td>
</tr>
<tr>
<td>$L_{ij}$</td>
<td>Transmission Line Inductance</td>
<td>2.5 mH</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Nominal Frequency</td>
<td>314.159 rad/s</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Nominal Voltage (peak)</td>
<td>150 V</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Cutoff Frequency - Droop Controller</td>
<td>2π rad/s</td>
</tr>
</tbody>
</table>
Figure 4.2: Experimental Microgrid Diagram.

Figure 4.3: Experimental Setup.
The weighting factors used by the cost function were heuristically tuned, considering the guidelines stated on section 3.1.4 considering the trade-off among the regulation and consensus objectives, and if required giving priority to one of the control objectives over the other objectives. The estimated delay, $\hat{\tau}_{ij}$, corresponds to one sampling period on the secondary level ($T_{sec} = 0.05 s$) and it is determined according to the delay requirement for control information shown in Table 4.2. The DMPC general parameters and weighting factors are shown in Table 4.3 and Table 4.4. The proportional-integral (PI) inner loops gains for current and voltage controllers are computed according with the guidelines presented in [71], considering the cut-off frequency separation between the nested loops. In this case the inner current loop was tuned to operate on 100 Hz, whereas the inner voltage loop operates on 10 Hz as the cut-off frequency. More details about the synthesis of the primary control level parameters are shown in appendix A2.

### Table 4.3: DMPC General Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{sec}$</td>
<td>Secondary Level Sampling Period</td>
<td>0.05 s</td>
</tr>
<tr>
<td>$\hat{\tau}_{ij}$</td>
<td>Estimated Communication Delay</td>
<td>0.05 s</td>
</tr>
<tr>
<td>$N_y$</td>
<td>Prediction Horizon</td>
<td>10</td>
</tr>
<tr>
<td>$N_u$</td>
<td>Control Horizon</td>
<td>10</td>
</tr>
<tr>
<td>$V_{max}$</td>
<td>Maximum Voltage</td>
<td>155V</td>
</tr>
<tr>
<td>$V_{min}$</td>
<td>Minimum Voltage</td>
<td>145V</td>
</tr>
</tbody>
</table>

### Table 4.4: DMPC Weighting Factors

<table>
<thead>
<tr>
<th>Weighting Factors</th>
<th>DG₁</th>
<th>DG₂</th>
<th>DG₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$ $[\text{rad}^2]$</td>
<td>Average Frequency Error</td>
<td>3E4</td>
<td>5E4</td>
</tr>
<tr>
<td>$\lambda_2$ $[\text{V}]^2$</td>
<td>Average Voltage Error</td>
<td>5E0</td>
<td>6E0</td>
</tr>
<tr>
<td>$\lambda_3$ $[\text{rad}^2]$</td>
<td>Frequency Control Action</td>
<td>8E4</td>
<td>8E4</td>
</tr>
<tr>
<td>$\lambda_4$ $[\text{V}]^2$</td>
<td>Voltage Control Action</td>
<td>5E3</td>
<td>5E3</td>
</tr>
<tr>
<td>$\lambda_5$ $[\text{VA}]^2$</td>
<td>Real Power Consensus</td>
<td>1.5E2</td>
<td>1.3E2</td>
</tr>
<tr>
<td>$\lambda_6$ $[\text{VAR}]^2$</td>
<td>Reactive Power Consensus</td>
<td>5E3</td>
<td>2E3</td>
</tr>
</tbody>
</table>

Four scenarios were implemented with the experimental setup using the proposed DMPC. The first scenario shows the DMPC performance when the microgrid is disturbed with load changes; it is considered as the base case and used for comparison purposes on the following
scenarios. In the second scenario, a communication failure between DG \(_1\) and DG \(_2\) is forced while the microgrid is disturbed. The third scenario evaluates the plug and play (PnP) capability, disconnecting and reconnecting the \(DG_3\) from/to the microgrid. Finally, the fourth scenario shows the microgrid performance when the latency changes over the communication network.

- **Test 4.1: Load changes (Base case).** This test analyzes the behavior using the proposed DMPC, when only load impacts are applied over the microgrid showed in Fig. 4.2 and preserving the communication network according with the adjacency matrix given by (4.22). In this case, the microgrid begins without load, and at \(t = 38s\), load \(Z_1\) is connected to the microgrid. At \(t = 58s\), the total load in the microgrid is increased, connecting also \(Z_2\). Finally at \(t = 78s\) and \(t = 98s\), the load is reduced to \(Z_2\) and zero, respectively.

In Fig. 4.4 and Fig. 4.5 it is shown that the average frequency and average voltage are regulated; however, voltage deviations over each DG caused by the microgrid heterogeneity are observed. Fig. 4.6 and Fig. 4.7 show that the consensus of active and reactive power are achieved, regarding that the power contribution of each DG is normalized according with its maximum capacity.

\[
A(t) = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \tag{4.22}
\]

![Figure 4.4](image_url)  
Figure 4.4: Frequency Regulation Against Load Changes - DMPC Base Case.
Figure 4.5: Voltage regulation against load changes - DMPC Base Case.

Figure 4.6: Active Power Consensus Against Load Changes - DMPC Base Case.

Figure 4.7: Reactive Power Consensus Against Load Changes - DMPC Base Case.
• **Test 4.2: Communication Path Failure.** This test uses the same load changes than the test 4.1, but adds a failure over the communication path between $DG_1$ and $DG_2$ at $t = 50s$. This failure is kept until the end of the test. This type of failure can be understood as a physical failure over the communication path or a sequence of data packet losses.

Despite the failure, the regulation of average frequency and average voltage is preserved, as well as the consensus objectives, because the adjacency matrix is automatically updated as a function of the received information on each DG. The microgrid response is shown in Fig. 4.8. This can be understood as a communication fault-tolerance feature of the proposed DMPC; however, as shown in Fig. 4.9, a difference in the transient state is observed when load changes are applied. It is because the adjacency terms are updated, but not the weighting factors, affecting the relationship between the adjacency matrix and the cost function stated with the tuning process, and using the whole communication network; then it is not possible to achieve the same transient compensation when the load changes. Note that the consensus responses for $DG_1$ and $DG_2$ are slower when the communication fails because it implies less information available to compensate the microgrid disturbances.

![Microgrid Frequency - DMPC Control](image)

![Microgrid Active Power - DMPC Control](image)

![Microgrid Direct Voltage - DMPC Control](image)

![Microgrid Reactive Power - DMPC Control](image)

Figure 4.8: Microgrid Response Against Communication Failure Between $DG_1$ and $DG_2$. 

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Figure 4.9: Consensus Detail - Microgrid Response Against Communication Failure Between DG1 and DG2. Zoom of Fig. 4.6 (top-left), Fig. 4.7 (bottom-left) and Fig. 4.8 (right).

- **Test 4.3: Plug-and-Play Capability.** In this test, the DG3 fails and is disconnected from the microgrid (at \( t \approx 49s \)); disabling its secondary controller. When the failure is cleared, a synchronization sequence is executed only on DG3, then, it is reconnected to the microgrid (at \( t \approx 75s \)) and its secondary controller re-enabled.

Fig. 4.10 shows the active and reactive power distribution according to the DGs connected to the microgrid along the events chain. As well as the test 4.2, the adjacency matrix, \( A(t) \), is updated based on the information received by each DG, therein adjusting the consensus and the average values in the optimization problem, when DG3 is disconnected and reconnected.

As it is shown in Fig. 4.10 even though the DG3 is connected to the microgrid between \( t \approx 75s \) and \( t \approx 78s \), the active and reactive power contributions of DG3 are not in consensus. In this period, DG3 is synchronized (\( \delta \theta_3 = 0 \)), but its secondary controller is disabled; thus, according to equations (3.3) and (4.2), only the reactive power flow through \( L_3 \) is feasible. When the secondary controller is enabled on DG3, the power consensus among the three units is re-established.

Fig. 4.11 shows the frequency and voltage responses along the test. Note that once the DG3 is disconnected from the microgrid, its frequency and voltage are the nominal values, and its output current is zero, as is shown in Fig. 4.12 because this DG does not have load, therefore its droop control leads DG3 to this operation point. When the DG3 is synchronized with the microgrid, the angle phase used by the primary level changes,
then an undershoot in the frequency response is caused. When the synchronization process is executed, the voltage on $DG_3$ is reduced in order to guard the DG stability; once the synchronization finishes successfully, the operation voltage is re-established to the nominal value and the DG is reconnected to the microgrid.

Figure 4.10: Real Power (top) and Reactive Power (bottom) Behavior - Plug and Play Test.

Figure 4.11: Frequency (top) and Voltage (bottom) Behavior - Plug and Play Test.
• Test 4.4: Latency Response. This scenario compares the microgrid response at different values of real delay, $\tau_{ij}$, on the communication network, but preserving the estimated latency, $\hat{\tau}_{ij}$, as one sampling period ($0.05s$) over all the paths of the network. Although it is reported that emergent communication technologies used in microgrids have a latency of less than 100 ms [7], the microgrid performance was evaluated using a 1.2s delay as a maximum latency (24 times the sampling period $T_{sec}$), regarding that issues such as the weather or maintenance frequency diminish the communication performance in rural/remote areas [32]. Although an heterogeneous communication network carries a different latency over each path, this test uses a worst-case scenario where all the paths have the same latency [2, 44]; whereas a test that includes different latency values will be discussed in section 4.3 where a more complex microgrid topology is used. The results for frequency regulation and real power consensus are shown in Fig. 4.13 and Fig. 4.14 respectively.

From the results, it is possible to state that, as well as the test 3.3, the microgrid response increases its overshoot and its settling time when the communication delay also increases; however, even, the delay is 20-times the sampling period, the control objectives are achieved in the microgrid.

The DMPC latency compensation capability, is related to the rolling horizon, the sampling period and the delay estimation $\hat{\tau}_{ij}$. However, there is a trade-off between the computational effort required to solve the optimization problem, and longer horizons or a shorter sampling period. Even when the optimization problem is solved based on delayed information from neighboring DGs, the rolling horizon scheme updates the control sequence each sampling period, compensating for latency effects even beyond the prediction horizon.
Figure 4.13: Microgrid Behavior With Communication Delays- Frequency Response for $\tau_{ij} = 0.25s$, $\tau_{ij} = 0.5s$, $\tau_{ij} = 1s$, and $\tau_{ij} = 1.2s$. 
Figure 4.14: Microgrid Behavior With Communication Delays- Active Power Response for $\tau_{ij} = 0.25s$, $\tau_{ij} = 0.5s$, $\tau_{ij} = 1s$, and $\tau_{ij} = 1.2s$. 
Although the proper microgrid performance using the proposed DMPC was evidenced, two additional tests were carried out to show that, when the DMPC scheme is used, the microgrid is easily scalable and additional advantages are achieved, compared with the DAPI scheme. The results of this tests are shown in Section 4.3.

4.3 Scalability and Comparison with DAPI controller: Simulation results

To discuss the scalability of the proposed DMPC, and to compare it with DAPI controller, a simulated microgrid with six DGs is built, using the simulation environment employed in chapter 3 and using the parameters of Table 4.1 and Table 4.3 in order to obtain results as close as possible to the experimental setup.

New generators $DG_4$, $DG_5$ and $DG_6$ were included with the same power capacities and droop slopes than $DG_3$, $DG_2$ and $DG_1$ respectively (Table 4.2), and the weighting factors used on the DMPC for the new DGs are shown in Table 4.5. Fig. 4.15 shows a diagram with the microgrid configuration used. Note that additional loads $Z_5 = Z_6 = (0.045 - 0.0637j)\Omega$ were included to increase the reactive power demand.

- **Test 4.5: Scalability.** In this test $DG_1$, $DG_2$, $DG_3$, as well as $Z_1$ and $Z_2$ are connected as initial condition; whereas $DG_4$, $DG_5$ and $DG_6$ were connected to the microgrid at $t = 20$, $t = 40$ and $t = 50$, respectively. Before to include the new DGs to the microgrid, a broadcast message, including only the new DG sub-index and its maximum apparent power, is sent. This information is required by the neighboring DGs in order to update the parameters $p$ (number of units on the microgrid), and $S_{jmax}$ (power capacity of the new DG), used in equations (4.21), (4.9), and (4.11). Note that this message does not compromise the PnP capability of the proposed DMPC, because the DG can be disconnected or re-connected (after the initial connection) without a re-transmission.
Table 4.5: DMPC Weighting Factors

<table>
<thead>
<tr>
<th>Weighting Factors</th>
<th>$DG_4$</th>
<th>$DG_5$</th>
<th>$DG_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1 \left[ \frac{\text{rad}}{s} \right]^2$ Average Frequency Error</td>
<td>9E4</td>
<td>5E4</td>
<td>3E4</td>
</tr>
<tr>
<td>$\lambda_2 \left[ \frac{1}{V} \right]^2$ Average Voltage Error</td>
<td>7E0</td>
<td>6E0</td>
<td>5E0</td>
</tr>
<tr>
<td>$\lambda_3 \left[ \frac{\text{rad}}{s} \right]^2$ Frequency Control Action</td>
<td>9E4</td>
<td>9E4</td>
<td>9E4</td>
</tr>
<tr>
<td>$\lambda_4 \left[ \frac{1}{V} \right]^2$ Voltage Control Action</td>
<td>5E3</td>
<td>5E3</td>
<td>5E2</td>
</tr>
<tr>
<td>$\lambda_5 \left[ \frac{\text{VA}}{W} \right]^2$ Active Power Consensus</td>
<td>4.0E2</td>
<td>6.5E2</td>
<td>2.5E2</td>
</tr>
<tr>
<td>$\lambda_6 \left[ \frac{\text{VAR}}{\text{VA}} \right]^2$ Reactive Power Consensus</td>
<td>5E3</td>
<td>7E3</td>
<td>7E3</td>
</tr>
</tbody>
</table>

In Fig. 4.16 are shown the active and reactive power responses when the new DGs are connected to the microgrid.

Figure 4.16: Active (Top) and Reactive (Bottom) Power Behavior - Simulated Six DGs Microgrid.
Test 4.6: Comparison with DAPI Controller. The DAPI scheme was introduced in chapter 2 and the discrete time control law (3.30) for frequency regulation and active power consensus using DAPI controller was discussed in chapter 3 in order to compare its performance to the proposed DMPC with the same control objectives. In this section the control law (3.30) is used again, and a new DAPI control law for voltage regulation and reactive power consensus is introduced as well. Both control laws are presented in equations (4.23), and (4.24), note that the term $P_{i_{\text{max}}}$ is replaced by $\frac{|S_{i_{\text{max}}}|}{|S_{j_{\text{max}}}|}$, in order to normalize the power contributions according with the total DG capacity. As explained in [3], $K_{i,\omega}$ and $K_{i,v}$ are the integral gains, whereas $\beta_i$, and $b_i$ weight the trade-off between voltage regulation and reactive power consensus (See the values on Table 4.6).

$$K_{i,\omega}s_i(t_n) = K_{i,\omega}s_i(t_{n-1}) - T_{\text{sec}}(\omega_i(t_n) - \omega_0) - T_{\text{sec}} \sum_{j=1,j\neq i}^p a_{ij}(t_n) \left( \frac{P_i(t_{n+k})}{|S_{i_{\text{max}}}|} - \frac{P_j(t_{n+k-\tau_{ij}})}{|S_{j_{\text{max}}}|} \right)$$  (4.23)

$$K_{i,v}V_{i,t_n} = K_{i,v}V_{i,t_{n-1}} - T_{\text{sec}}\beta_i(V_i(t_n) - V_0) - T_{\text{sec}} \sum_{j=1,j\neq i}^p b_{ij}(t_n) \left( \frac{Q_i(t_{n+k})}{|S_{i_{\text{max}}}|} - \frac{Q_j(t_{n+k-\tau_{ij}})}{|S_{j_{\text{max}}}|} \right)$$  (4.24)

**Table 4.6: DAPI Parameters**

<table>
<thead>
<tr>
<th>DAPI Parameters</th>
<th>DG1</th>
<th>DG2</th>
<th>DG3</th>
<th>DG4</th>
<th>DG5</th>
<th>DG6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{i,\omega}$</td>
<td>1.08E-1</td>
<td>1.04E-1</td>
<td>8E-1</td>
<td>8E-1</td>
<td>1.04E-1</td>
<td>1.08E-1</td>
</tr>
<tr>
<td>$K_{i,v}$</td>
<td>1E0</td>
<td>1E0</td>
<td>1E0</td>
<td>1E0</td>
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<td>1E0</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>5E-1</td>
<td>1E-1</td>
<td>9.5E-1</td>
<td>4E-1</td>
<td>1E-1</td>
<td>1E-1</td>
</tr>
<tr>
<td>$b_i$</td>
<td>9E0</td>
<td>1.05E1</td>
<td>7.5E0</td>
<td>1.2E1</td>
<td>1.35E1</td>
<td>1.35E1</td>
</tr>
</tbody>
</table>

A comparison between the proposed DMPC and the DAPI is justified because both schemes include regulation and consensus objectives, as well as, the communication network topology from the adjacency matrix. The comparison is focused on the reactive power consensus objective and the tolerance of higher latency in the communication network.

The voltage and reactive power responses are shown in Fig. 4.17 and Fig. 4.18 respectively. Note that, when $Z_5$ and $Z_6$ are connected to the microgrid, the voltages at the DGs output are similar for both schemes; however, the reactive power consensus is just achieved when the DMPC is used. Although it is possible to achieve the reactive power consensus using DAPI scheme as well, it implies a poor response for voltage regulation [3].
Figure 4.17: Voltage Response Using DMPC (Top) and DAPI (Bottom) Controllers.

Figure 4.18: Reactive Power Response Using DMPC (Top) and DAPI (Bottom) Controllers.
In Fig. 4.19 and Fig. 4.20, the real and reactive power responses are shown for when the microgrid is disturbed connecting $Z_6$ and applying a delay of $\tau_{ij} = 0.5s$ on all communication paths. As was shown in the test 4.2, DMPC compensates the delay effects by considering the delay estimation, $\hat{\tau}_{ij}$, and the rolling horizon into the optimization problem. Unlike the DMPC, the DAPI controller does not include a delay compensation mechanism in its control law; thus, the latency effect is evidenced as a more oscillatory response than the DMPC at the same value of $\tau_{ij}$. In this case, the test considers $Z_1$ and $Z_2$, as well as the six DGs, connected to the microgrid as initial condition. Note that although for both controllers the transient state is affected when the load changes, the proposed DMPC achieves the real and reactive power consensus; however, DAPI controller loses this feature, presenting oscillations for DG1 and DG6. In this case frequency and voltage regulation objectives do not present significant differences when DMPC or DAPI are used; it is because DMPC compensates the delay, whereas DAPI scheme achieve these objectives based only on local measurements. Frequency and voltage responses for both control schemes are presented in Fig. 4.21 and Fig. 4.22 respectively.

Figure 4.19: Power Behavior Using DMPC With Communication Delay $\tau_{ij} = 0.5s$. 

![Figure 4.19](image_url)
Figure 4.20: Power Behavior Using DAPI With Communication Delay $\tau_{ij} = 0.5s$.

Figure 4.21: Frequency Behavior Using DMPC and DAPI Schemes With Communication Delay $\tau_{ij} = 0.5s$. 
Figure 4.22: Voltage Behavior Using DMPC and DAPI Schemes With Communication Delay $\tau_{ij} = 0.5s$.

Different latency over each path is a possible scenario, especially when the technology used requires line of sight or is susceptible to interference. Note that equations (4.21), (4.9), and (4.11) associate an independent estimation, $\hat{\tau}_{ij}$, for each $DG_j$ that shares information with $DG_i$. Fig. 4.23 and Fig. 4.24 present the microgrid response when different delays are used in the network, using DMPC scheme; whereas the delay values used in this test are included in Table 4.7.

Table 4.7: Utilized Values in Simulation with Different Delays

<table>
<thead>
<tr>
<th>Communication Delay</th>
<th>$\tau_{12}$</th>
<th>$\tau_{13}$</th>
<th>$\tau_{16}$</th>
<th>$\tau_{23}$</th>
<th>$\tau_{34}$</th>
<th>$\tau_{45}$</th>
<th>$\tau_{56}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay [s]</td>
<td>0.15</td>
<td>0.1</td>
<td>0.75</td>
<td>0.2</td>
<td>0.25</td>
<td>0.35</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The results from Fig. 4.23 and Fig. 4.24 shown that both, regulation and consensus objectives are achieved, However, the oscillations in the consensus responses are lead by $DG_6$, which is affected by the two larger delays on the network ($\tau_{16}$ and $\tau_{56}$). It means that a scenario where $\tau_{ij} = \tau_{\text{max}}, \forall i, j \in \{1, ..., p\}$, where $\tau_{\text{max}}$ is the maximum delay in the network, as it was considered in previous scenarios, will present the worst possible microgrid response, because all controllers solve the optimization based on information delayed $\tau_{\text{max}}$. 

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Figure 4.23: Frequency and Voltage Behavior With Different Communication Delays

Figure 4.24: Active and Reactive Power Behavior With Different Communication Delays
4.4 Discussion

In this chapter a DMPC which includes frequency and voltage regulation, as well as active and reactive power consensus, as control objectives, was proposed. The model used in this controller considers the co-dependency among the microgrid variables and establishes new operational constraints, such as voltage and apparent power ranges, to preserve the microgrid behavior within a feasible solution space; however, to preserve the QP structure with linear constraints, active, reactive and apparent power expressions were linearized around the measured point, allowing to update the prediction model for every sampling period.

It is a remarkable advantage that the optimization problem is updated every sampling period, not only by the local measurements, but by information from neighboring DGs, achieving the control objectives even when both the microgrid, and communication network topologies change.

The proposed controller was deployed in an experimental setup, validating that the proposed optimization problem is solved according to the secondary control requirements. The experimental results shown a properly performance when electrical and communication issues, such as strong load impacts, extended data dropouts, disconnection and reconnection of DGs, and higher latency, disturb the microgrid.

Additional simulation tests were performed evidencing the microgrid behavior when new DGs are integrated to the microgrid, and showing that, unlike DAPI scheme, using the proposed DMPC, average voltage regulation and reactive power consensus are achievable simultaneously, even when large delays affect the information sharing.
Chapter 5

Conclusions and Final Remarks

From the literature review, it has been stated that distributed schemes for secondary control level in microgrids is an emergent research topic. Several schemes based on PI controllers have been reported, not only for frequency and voltage regulation, but including consensus of active and reactive power in the microgrid. This consensus feature has improving the microgrid stability, ensuring an equitable power distribution among the DGs. As it was shown, frequency regulation and active power consensus are achievable simultaneously, regarding that frequency is a universal parameter in the microgrid; it means that in steady state only one frequency value exist in the microgrid. However, using PI and consensus based controllers, the voltage regulation and reactive power consensus imply a trade-off, which should be determined according to the microgrids features.

Another approach used in secondary control is known as finite time control. In this approach, non-linear control laws are stated to ensure the control objectives convergence. Using finite time controllers, frequency and voltage consensus are included as additional objectives in order to brings the leader role to one DG in the microgrid. Predictive control has been also reported for secondary level in microgrids; although DMPC provides better tools, than PI based and finite time controllers, to attend electrical and communication issues, the main obstacle to implement this type of controllers is the computational effort to solve the optimization problem, regarding that this should be stated using local measurements, and global information from the microgrid.

In chapter 3 a DMPC for frequency regulation and power consensus was presented. The proposed non-iterative DMPC, uses as input local measurements, to compute the droop effects and the active power contribution, and also the output vector from neighboring DGs to compute the control action required, achieving the control objectives. The constraints used in the QP problem bound the feasible solution space, reducing the computational effort, and being possible to deploy the controller, satisfying the requirements of secondary control level, such as the settling time. Furthermore, the proposed DMPC for frequency regulation shows a better performance than the DAPI scheme for communication issues, such as network unavailability or long latency.

In chapter 4 an extension of the previously described DMPC, is presented. This case
includes two additional equations to model the voltage-reactive power droop, and the reactive power contribution, in order to consider the co-dependent behavior of these variables and including as control objectives the average voltage regulation and the reactive power consensus. Although the optimization problem is more complex than the previous case, it is able to be deployed in the experimental setup composed of three DGs. In this case, a better performance than the DAPI scheme, when the microgrid is disturbed by communication issues.

The structure proposed for both controllers allows the plug and play capability in the microgrid, improving the robustness when both, electrical and communication topologies change. This feature, also permits that the microgrid be scalable, adding new DGs without a significant rise of the optimization complexity. Furthermore, the proposed communication model, based on the received information for each DG, permits to attend the data drop-outs as changes in the communication topology, considering only the available information in the optimization problem; then in a worst case scenario, where the whole communication network is unavailable, the optimization is solved in a decentralized manner.

5.1 Future Work

Although the proposed control schemes are novel, some research lines can be proposed:

- **DMPC for Hybrid AC/DC Microgrids:** This extension consists of to modify the optimization problem to use voltage regulation and active power consensus as control objectives. Considering that droop, and power transfer equations are modelled, as well as operational constraints, to control AC and DC generators with the proposed structure require minor concerns. However, an additional optimization problem should be stated to control the AC and DC power flow over the interlinking converters.

- **DMPC for Unbalances Control:** Although the proposed DMPC in chapter 4 includes reactive power consensus, it is managed over a balanced microgrid. However, an additional extension can be proposed considering that electrical unbalances imply different reactive power contribution over each phase. Then, including a power contribution model for each phase, and adding reactive power consensus in the cost function, it is possible to mitigate the unbalances effects in the microgrid.

- **Stability Concerns:** In this thesis the stability analysis is not pursued. The stability concerns covered in this work are focused on to attend feasibility issues about microgrid operation, e.g. preserving the power contribution within the feasible range on each DG. However, to ensure stability on predictive control, requires to demonstrate recursive and bounded feasibility, being it an open problem with the proposed structure.

- **Communication Concerns:** Although networking and automation protocols were reviewed in this thesis, communication issues such as latency, data dropouts and changes on topology covered from a control point-of-view, can be used as an input to state requirements and changes about communication protocols developed and technologies (wired or wireless) used in microgrids.
Appendices
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSI</td>
<td>Current Source Inverter</td>
</tr>
<tr>
<td>DAPI</td>
<td>Distributed Averaging Proportional-Integral control</td>
</tr>
<tr>
<td>DG</td>
<td>Distributed Generator</td>
</tr>
<tr>
<td>DMPC</td>
<td>Distributed Model-based Predictive Control</td>
</tr>
<tr>
<td>DNP</td>
<td>Distributed Network Protocol</td>
</tr>
<tr>
<td>IoT</td>
<td>Internet of Things</td>
</tr>
<tr>
<td>IP</td>
<td>Internet Protocol</td>
</tr>
<tr>
<td>M2M</td>
<td>Machine to Machine</td>
</tr>
<tr>
<td>MPC</td>
<td>Model-based Predictive Control</td>
</tr>
<tr>
<td>MPPT</td>
<td>Maximum Power Point Tracker</td>
</tr>
<tr>
<td>OPC</td>
<td>OLE (Object Linking and Embedding) for Process Control</td>
</tr>
<tr>
<td>OSI/ISO</td>
<td>Open System Interconnection/International Standard Organization</td>
</tr>
<tr>
<td>PI</td>
<td>Proportional-Integral control</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-Integral-Derivative control</td>
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<tr>
<td>PLC</td>
<td>Power Line communication</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase Locked Loop</td>
</tr>
<tr>
<td>PMU</td>
<td>Phasor Measurement Unit</td>
</tr>
<tr>
<td>PnP</td>
<td>Plug and Play</td>
</tr>
<tr>
<td>PSO</td>
<td>Particle Swarm Optimization</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>QP</td>
<td>Quadratic Programming</td>
</tr>
<tr>
<td>RTT</td>
<td>Real Time Target</td>
</tr>
<tr>
<td>SDN</td>
<td>Software Defined Network</td>
</tr>
<tr>
<td>STP</td>
<td>Shielded Twisted Pair</td>
</tr>
<tr>
<td>TCP</td>
<td>Transmission Control Protocol</td>
</tr>
<tr>
<td>UTP</td>
<td>Unshielded Twisted Pair</td>
</tr>
<tr>
<td>VPN</td>
<td>Virtual Private Network</td>
</tr>
<tr>
<td>VSI</td>
<td>Voltage Source Inverter</td>
</tr>
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</table>
A2 Primary Inner Loops Design

In this Appendix the design process for the current and voltage inner loops, used on each DG for simulation and experimental purposes, is described. The primary control level considers a rotative dq framework oriented with the voltage vector at the LC filter output. Both control loops are nested, therefore a cut-off frequency separation is mandatory between them in order to avoid dynamical coupling. The angle phase, \( \theta_e \), required to rotate the reference framework is computed using a phase locked loop (PLL) at the DG output. A control diagram for inner controllers is shown in Fig. A2.1.

Figure A2.1: Control Scheme for Inner Current and Voltage Loops in a dq Framework. Figure taken from [5].

To determine the controller gains, the guidelines established on [71] are followed. The inner current loop is assumed as the faster loop, and the plant transfer function used to design the controller is defined only by the filter inductance \( L_{fi} \). The differential equations used to tuning purposes of current controllers in direct and quadrature axes are presented in (A2.1) and (A2.2).

\[
v_{di}(t) = -\omega_0 L_{fi} i_lq(t) + K_{pc}(i_d^*(t) - i_d(t)) + K_{ic} \int (i_d(t)^* - i_d(t))dt \]  
(A2.1)

\[
v_{qi}(t) = \omega_0 L_{fi} i_{ld}(t) + K_{pc}(i_q^*(t) - i_q(t)) + K_{ic} \int (i_q(t)^* - i_q(t))dt \]  
(A2.2)

For voltage control purposes, the same procedure is followed. Because this closed loop is considered slower than the current loop, the plant is defined only considering the filter capacitance \( C_{fi} \). The differential equations stated for the voltage loop are presented in (A2.3), and (A2.4). Note that the voltage controllers output, \( i_d^* \) and \( i_q^* \), are used as set points by the current loops, whereas the current controllers output correspond to the voltage inputs \( v_{di} \) and \( v_{qi} \), required by the power electronics converter.
\[ i_{td}^*(t) = -\omega_0 C_{fi} v_{qo}(t) + K_{pv} (v^*_{do}(t) - v_{do}(t)) + K_{iv} \int (v_{do}(t)^* - v_{do}(t)) \, dt \] (A2.3)

\[ i_{tq}^*(t) = \omega_0 C_{fi} v_{do}(t) + K_{pv} (v^*_{qo}(t) - v_{qo}(t)) + K_{iv} \int (v_{qo}(t)^* - v_{qo}(t)) \, dt \] (A2.4)

The cut-off frequency is determined by the controller proportional and integral gains, \( K_{pc} \) and \( K_{ic} \) for current loop, and \( K_{pv} \) and \( K_{iv} \) for voltage loop; and these can be determined by typical tuning methods for PID controllers. After a discretization of differential equations (A2.1), (A2.2), (A2.3), and (A2.4), the pole placement method is used to tune the controllers gains. The parameters used, and the tuning results are shown in Table A2.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{prim} )</td>
<td>Primary Level Sampling Period</td>
<td>1/16E3 s</td>
</tr>
<tr>
<td>( L_{fi} )</td>
<td>Filter Inductance</td>
<td>0.85 mH</td>
</tr>
<tr>
<td>( C_{fi} )</td>
<td>Filter Capacitance</td>
<td>70 ( \mu )F</td>
</tr>
<tr>
<td>( \omega_0 )</td>
<td>Nominal Frequency</td>
<td>314.159 rad/s</td>
</tr>
<tr>
<td>( V^*_{do} )</td>
<td>Direct Voltage Set Point</td>
<td>150 V</td>
</tr>
<tr>
<td>( V^*_{qo} )</td>
<td>Quadrature Voltage Set Point</td>
<td>0 V</td>
</tr>
<tr>
<td>( \omega_{cc} )</td>
<td>Cutoff Frequency - Current Loop</td>
<td>628.3 rad/s</td>
</tr>
<tr>
<td>( K^*_{pc} )</td>
<td>Proportional Gain - Current Loop</td>
<td>5.341 V/A</td>
</tr>
<tr>
<td>( K^*_{ic} )</td>
<td>Integral Gain - Current Loop</td>
<td>8171 V/As</td>
</tr>
<tr>
<td>( \omega_{cv} )</td>
<td>Cutoff Frequency - Voltage Loop</td>
<td>62.83 rad/s</td>
</tr>
<tr>
<td>( K^*_{pv} )</td>
<td>Proportional Gain - Voltage Loop</td>
<td>0.441 A/V</td>
</tr>
<tr>
<td>( K^*_{iv} )</td>
<td>Integral Gain - Voltage Loop</td>
<td>69.67 A/Vs</td>
</tr>
</tbody>
</table>
A3  Discretization and Linearization of Electrical Model

In this section are detailed the discretization and linearization procedures for the electrical model stated by (3.1), (3.2), and (3.3) for DMPC for frequency regulation in chapter 3; and complemented by (4.1), and (4.2) for DMPC for frequency and voltage regulation proposed in chapter 4. The procedure uses forward Euler method, defined by (A3.1) to discretize all equations, whereas Taylor expansion, around the point $P = \{x_1, x_2, ..., x_n\}$, is defined by (A3.2) and used to linearize equations (3.3) and (4.2). In this case is considered $t_n = nT_{sec}, n \in \mathbb{Z}^+$, and $T_{sec}$ as the sampling period used in secondary control level.

\begin{equation}
T_{sec}\frac{df(t)}{dt}\bigg|_{t=t_n} = [f(t_{n+1}) - f(t_n)] \tag{A3.1}
\end{equation}

\begin{align*}
f(y_1, y_2, ..., y_n) &\approx f(\mathbb{P}) + \frac{\partial f(y_1, y_2, ..., y_n)}{\partial y_1}\bigg|_{\mathbb{P}} (y_1 - x_1) + \frac{\partial f(y_1, y_2, ..., y_n)}{\partial y_2}\bigg|_{\mathbb{P}} (y_2 - x_2) \\
&+ ... + \frac{\partial f(y_1, y_2, ..., y_n)}{\partial y_n}\bigg|_{\mathbb{P}} (y_n - x_n) \tag{A3.2}
\end{align*}

A3.1 Droop Equations

The linearization process is shown from frequency droop equation (3.1) to obtain (3.4) and (4.3). The same procedure is applied to voltage droop equation (4.1) to obtain (4.4).

Equation (3.1) is rewritten as (A3.3), and it is possible to rewrite $\omega_{s,i}(t)$ in terms of $\Delta \omega_{s,i}(t)$.

\begin{equation}
\omega_i(t) = \omega_0 + M_{\omega,i}P_i(t) + \omega_{s,i}(t) \tag{A3.3}
\end{equation}

\begin{equation}
\omega_i(t) = \omega_0 + M_{\omega,i}P_i(t) + \frac{1}{T_{sec}} \int \Delta \omega_{s,i}(t) dt
\end{equation}

Deriving both sides and applying forward Euler method:

\begin{equation}
\frac{d\omega_i(t)}{dt} = M_{\omega,i} \frac{dP_i(t)}{dt} + \frac{1}{T_{sec}} \Delta \omega_{s,i}(t) \tag{A3.4}
\end{equation}
A3.2 Phase Angle Equation

There is shown the linearization process from (3.2) to (3.5) and (4.5), using two procedures with the same result. Equation (3.2) is rewritten as (A3.5).

\[
\delta \theta_i(t) = \theta_i(t) - \theta_i^*(t) = \int_0^t \left[ \omega_i(\tau) - \omega_i^*(\tau) \right] d\tau \quad \text{(A3.5)}
\]

- **Procedure 1:**

\[
\delta \theta_i(t) = \int_0^t \left[ \omega_i(\tau) - \omega_i^*(\tau) \right] d\tau
\]

Deriving both sides and applying forward Euler method

\[
\left. \frac{d\delta \theta_i(t)}{dt} \right|_{t=t_n} = \frac{d}{dt} \left[ \int_0^t \left[ \omega_i(\tau) - \omega_i^*(\tau) \right] d\tau \right] \bigg|_{t=t_n}
\]

\[
\delta \theta_i(t_{n+1}) - \delta \theta_i(t_n) = \int_0^{t_{n+1}} \left[ \omega_i(\tau) - \omega_i^*(\tau) \right] d\tau - \int_0^{t_n} \left[ \omega_i(\tau) - \omega_i^*(\tau) \right] d\tau
\]

\[
\delta \theta_i(t_{n+1}) - \delta \theta_i(t_n) = \int_{t_n}^{t_{n+1}} \left[ \omega_i(\tau) - \omega_i^*(\tau) \right] d\tau
\]

\[
\delta \theta_i(t_{n+1}) = \delta \theta_i(t_n) + T_{sec}[\omega_i(t_n) - \omega_i^*(t_n)] \quad \text{(A3.6)}
\]

- **Procedure 2:** Considering

\[
\theta_i(t) = \int_0^t \omega_i(\tau), d\tau
\]

Deriving both sides and applying forward Euler method

\[
\left. \frac{d\theta_i(t)}{dt} \right|_{t=t_n} = \frac{d}{dt} \left[ \int_0^t \omega_i(\tau), d\tau \right] \bigg|_{t=t_n}
\]

\[
\theta_i(t_{n+1}) - \theta_i(t_n) = \int_0^{t_{n+1}} \omega_i(\tau) d\tau - \int_0^{t_n} \omega_i(\tau) d\tau
\]

\[
\theta_i(t_{n+1}) - \theta_i(t_n) = \int_{t_n}^{t_{n+1}} \omega_i(\tau), d\tau
\]

\[
\theta_i(t_{n+1}) - \theta_i(t_n) = T_{sec}\omega_i(t_n)
\]

Then, from (A3.5)

\[
\delta \theta_i(t_{n+1}) = \theta_i(t_{n+1}) - \theta_i^*(t_{n+1})
\]

\[
\delta \theta_i(t_{n+1}) = [\theta_i(t_n) + T_{sec}\omega_i(t_n)] - [\theta_i^*(t_n) + T_{sec}\omega_i^*(t_n)]
\]

Re-ordering the terms

\[
\delta \theta_i(t_{n+1}) = [\theta_i(t_n) - \theta_i^*(t_n)] + [T_{sec}\omega_i(t_n) - T_{sec}\omega_i^*(t_n)]
\]

\[
\delta \theta_i(t_{n+1}) = \delta \theta_i(t_n) + T_{sec}[\omega_i(t_n) - \omega_i^*(t_n)] \quad \text{(A3.7)}
\]
A3.3 Power Transfer Equations

Because power transfer equations \((3.3)\) and \((4.2)\) are non-linear, these are linearized using Taylor expansion around the measured point \(P(t_n) = \{\omega_i(t_n), \omega_i^*(t_n), V_i(t_n), V_i^*(t_n), \delta \theta_i(t_n), P_i(t_n), Q_i(t_n)\}\). After that, forward Euler discretization is applied to the linearized equations. The same procedure is used to both equations, \((3.3)\) and \((4.2)\) to obtain \((3.6)\), \((4.6)\) and \((4.7)\). Equation \((3.3)\) is rewritten as \((A3.8)\), and only the procedure to obtain \((3.6)\) and \((4.6)\) is shown below.

\[
P_i(t) = B_i V_i(t) V_i^*(t) \sin(\delta \theta_i(t))
\]

(A3.8)

Linearizing

\[
P_i(t) = P_i(t_n) + \left. \frac{\partial P_i(t)}{\partial V_i} \right|_{P(t_n)} [V_i(t) - V_i(t_n)] + \left. \frac{\partial P_i(t)}{\partial V_i^*} \right|_{P(t_n)} [V_i^*(t) - V_i^*(t_n)] + \left. \frac{\partial P_i(t)}{\partial \delta \theta_i} \right|_{P(t_n)} [\delta \theta_i(t) - \delta \theta_i(t_n)]
\]

\[
P_i(t) = P_i(t_n) + K_V [V_i(t) - V_i(t_n)] + K_{V^*} [V_i^*(t) - V_i^*(t_n)] + K_{\delta \theta} [\delta \theta_i(t) - \delta \theta_i(t_n)]
\]

Where

\[
K_V = B_i V_i^*(t_n) \sin(\delta \theta_i(t_n))
\]

\[
K_{V^*} = B_i V_i(t_n) \sin(\delta \theta_i(t_n))
\]

\[
K_{\delta \theta} = B_i V_i(t_n) V_i^*(t_n) \cos(\delta \theta_i(t_n))
\]

Deriving both sides and evaluating at \(t = t_n\)

\[
\left. \frac{dP_i(t)}{dt} \right|_{t=t_n} = K_V \left. \frac{dV_i(t)}{dt} \right|_{t=t_n} + K_{V^*} \left. \frac{dV_i^*(t)}{dt} \right|_{t=t_n} + K_{\delta \theta} \left. \frac{d\delta \theta_i(t)}{dt} \right|_{t=t_n}
\]

\[
P_i(t_{n+1}) - P_i(t_n) = K_V [V_i(t_{n+1}) - V_i(t_n)] + K_{V^*} [V_i^*(t_{n+1}) - V_i^*(t_n)] + K_{\delta \theta} [\delta \theta_i(t_{n+1}) - \delta \theta_i(t_n)]
\]

Assuming \(V^*\) as a constant, \((4.6)\) is obtained, and it is used on chapter 4 for DMPC for frequency and voltage regulation, where \(P, V, \) and \(\delta \theta\) are included as optimization variables:

\[
V_i^*(t_{n+1}) - V_i^*(t_n) = 0
\]
\[ P_i(t_{n+1}) = P_i(t_n) + K_V [V_i(t_{n+1}) - V_i(t_n)] + K_{\delta \theta} [\delta \theta_i(t_{n+1}) - \delta \theta_i(t_n)] \]

Then
\[
P_i(t_{n+1}) = P_i(t_n) + [B_i V_i^* (t_n) \sin(\delta \theta_i(t_n))][V_i(t_{n+1}) - V_i(t_n)]
+ [B_i V_i(t_n) V_i^*(t_n) \cos(\delta \theta_i(t_n))][\delta \theta_i(t_{n+1}) - \delta \theta_i(t_n)]
\] (A3.9)

Finally, also assuming \( V \) as a constant, [3.6], used on chapter 3 for DMPC for frequency regulation, is obtained:
\[
P_i(t_{n+1}) = P_i(t_n) + [B_i V_i(t_n) V_i^*(t_n) \cos(\delta \theta_i(t_n))][\delta \theta_i(t_{n+1}) - \delta \theta_i(t_n)]
\] (A3.10)

\[
\begin{align*}
|S_i(t)| &= (P_i(t)^2 + Q_i(t)^2)^{1/2} < S_{\text{max}} 
\end{align*}
\] (A3.11)

Linearizing \( |S_i(t)| \) around the measured point \( \mathbb{P} \):
\[
\begin{align*}
|S_i(t)| &\approx (P_i(t_n)^2 + Q_i(t_n)^2)^{1/2} + \frac{\partial |S_i(t)|}{\partial P_i}(P_i(t) - P_i(t_n)) + \frac{\partial |S_i(t)|}{\partial Q_i}(Q_i(t) - Q_i(t_n)) \\
&\approx (P_i(t_n)^2 + Q_i(t_n)^2)^{1/2} + \frac{1}{2} (P_i(t_n)^2 + Q_i(t_n)^2)^{-1/2} (2P_i(t_n))(P_i(t) - P_i(t_n)) \\
&\quad + \frac{1}{2} (P_i(t_n)^2 + Q_i(t_n)^2)^{-1/2} (2Q_i(t_n))(Q_i(t) - Q_i(t_n))
\end{align*}
\] (A3.12)

[A3.12] is not feasible when \( P_i(t_n) = Q_i(t_n) = 0 \); therefore, applying the triangular inequality the polytopic inner approximation [A3.1] is achieved.
\[
(P_i(t)^2 + Q_i(t)^2)^{1/2} < |P_i(t)| + |Q_i(t)| = f(P_i, Q_i)
\] (A3.13)

Note that, if the right side of inequality is less than \( S_{\text{max}} \), the left side will also be. Therefore, linearizing:
\[
\begin{align*}
|P_i(t)| + |Q_i(t)| &\approx |P_i(t_n)| + |Q_i(t_n)| + \frac{\partial f(P_i, Q_i)}{\partial P_i}(P_i(t) - P_i(t_n)) + \frac{\partial f(P_i, Q_i)}{\partial Q_i}(Q_i(t) - Q_i(t_n)) \\
&\approx |P_i(t_n)| + |Q_i(t_n)| + \text{sign}(P_i(t_n))(P_i(t) - P_i(t_n)) \\
&\quad + \text{sign}(Q_i(t_n))(Q_i(t) - Q_i(t_n))
\end{align*}
\] (A3.14)
In this case the discretization can be directly derived; then, extending along the prediction horizon, (A3.15) can be stated.

\[
|P_i(t_n)| + |Q_i(t_n)| + \text{sign}(P_i(t_n))(P_i(t + k) - P_i(t_n)) \\
+ \text{sign}(Q_i(t_n))(Q_i(t + k) - Q_i(t_n)) < S_{max}
\]  

(A3.15)
A4 QP Problem Synthesis

In this section, the QP problem synthesis, from the cost function (3.16), the model composed of equations (3.10), (3.11), (3.12), and operational constraints (3.13), (3.15), (3.14), is covered. This procedure proceeds for the DMPC for frequency regulation presented on chapter 3, but it can be extended to the synthesis of the QP problem for DMPC for frequency and voltage regulation on chapter 4.

To synthesize the matrices required to the QP problem, the optimization vector \(X_i\) defined by (3.18) is rewrote as (A4.1).

\[
X_i = [\bar{\omega}_i(t_{n+1}), \ldots, \bar{\omega}_i(t_{n+N_y}), \Delta \omega_{s,i}(t_n), \ldots, \Delta \omega_{s,i}(t_{n+N_u-1}), \omega_i(t_{n+1}), \ldots, \omega_i(t_{n+N_y}), \delta \theta_i(t_{n+1}), \ldots, \delta \theta_i(t_{n+N_y}), P_i(t_{n+1}), \ldots, P_i(t_{n+N_y})]^T
\] (A4.1)

- **Cost function**: The cost function (3.16) is rewrote as (A4.2).

\[
J_i(t_n) = \sum_{k=1}^{N_u} \lambda_{1i}(\bar{\omega}_i(t_{n+k}) - \omega_0)^2 + \sum_{k=1}^{N_u} \lambda_{2i}((\Delta \omega_{s,i}(t_{n+k-1}))^2
\]
\[
+ \sum_{j=1, j \neq i}^{P} \sum_{k=1}^{N_u} \lambda_{3i} a_{ij}(t_n) \left( \frac{P_i(t_{n+k})}{P_{i\text{ max}}} - \frac{P_j(t_{n+k-\tau_{ij}})}{P_{j\text{ max}}} \right)^2
\]

(A4.2)

Expanding the sums over the prediction and control horizons:

\[
J_i(t_n) = \lambda_{1i}[(\bar{\omega}_i(t_{n+1}) - \omega_0)^2 + (\bar{\omega}_i(t_{n+2}) - \omega_0)^2 + \cdots + (\bar{\omega}_i(t_{n+N_y}) - \omega_0)^2]
\]
\[
+ \lambda_{2i}[(\Delta \omega_{s,i}(t_n))^2 + (\Delta \omega_{s,i}(t_{n+1}))^2 + \cdots + (\Delta \omega_{s,i}(t_{n+N_u-1}))^2]
\]
\[
+ \lambda_{3i} \left[ \sum_{j=1, j \neq i}^{P} a_{ij}(t_n) \left( \frac{P_i(t_{n+1})}{P_{i\text{ max}}} - \frac{P_j(t_{n+1-\tau_{ij}})}{P_{j\text{ max}}} \right)^2 \right]
\]

Expanding the quadratic terms:

\[
J_i(t_n) = \lambda_{1i}[\bar{\omega}_i(t_{n+1})^2 + \cdots + \bar{\omega}_i(t_{n+N_y})^2 - 2\bar{\omega}_i(t_{n+1})\omega_0 - \cdots - 2\bar{\omega}_i(t_{n+N_y})\omega_0 + N_y\omega_0^2]
\]
\[
+ \lambda_{2i}[(\Delta \omega_{s,i}(t_n))^2 + \cdots + (\Delta \omega_{s,i}(t_{n+N_u-1}))^2]
\]
\[
+ \lambda_{3i} \left[ \sum_{j=1, j \neq i}^{P} a_{ij}(t_n) \frac{P_i^2(t_{n+1})}{P_{i\text{ max}}} \right] + \cdots + \sum_{j=1, j \neq i}^{P} a_{ij}(t_n) \frac{P_j^2(t_{n+N_y})}{P_{j\text{ max}}} \right]
\]
\[
- 2 \left[ \sum_{j=1, j \neq i}^{P} a_{ij}(t_n) \frac{P_i(t_{n+1}) P_j(t_{n+1-\tau_{ij}})}{P_{i\text{ max}} P_{j\text{ max}}} \right] + \cdots + \sum_{j=1, j \neq i}^{P} a_{ij}(t_n) \frac{P_i(t_{n+N_y}) P_j(t_{n+N_y-\tau_{ij}})}{P_{i\text{ max}} P_{j\text{ max}}} \right]
\]
\[
+ \left[ \sum_{j=1, j \neq i}^{P} a_{ij}(t_n) \frac{P_j^2(t_{n+1-\tau_{ij}})}{P_{j\text{ max}}} \right] + \cdots + \sum_{j=1, j \neq i}^{P} a_{ij}(t_n) \frac{P_j^2(t_{n+N_y-\tau_{ij}})}{P_{j\text{ max}}} \right]
\]
Reordering in a matrix notation, and regarding that \( P_j \) and \( \omega_0 \) are known, and constant, along the whole prediction horizon, it is possible to remove constant terms from the optimization, as follows:

\[
\min_{\bar{X}_i} \{ J_i(t_n) \} = \\
\min_{\bar{X}_i} \left\{ \lambda_{1i} \begin{bmatrix} \bar{w}_i(t_{n+1}) \\ \vdots \\ \bar{w}_i(t_{n+N_y}) \end{bmatrix} \Pi_{N_y} \begin{bmatrix} \bar{w}_i(t_{n+1}) \\ \vdots \\ \bar{w}_i(t_{n+N_y}) \end{bmatrix} - 2\lambda_{1i}\omega_0 \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_{1 \times N_y} \begin{bmatrix} \bar{w}_i(t_{n+1}) \\ \vdots \\ \bar{w}_i(t_{n+N_y}) \end{bmatrix} \\
+ \lambda_{2i} \begin{bmatrix} \Delta \omega_{s,i}(t_n) \\ \vdots \\ \Delta \omega_{s,i}(t_{n+N_u-1}) \end{bmatrix} \Pi_{N_u} \begin{bmatrix} P_t(t_{n+1}) \\ \vdots \\ P_t(t_{n+N_y}) \end{bmatrix} \\
+ \lambda_{3i} \frac{\Gamma_i(t_n)}{P_i^{max}} \begin{bmatrix} P_t(t_{n+1}) \\ \vdots \\ P_t(t_{n+N_y}) \end{bmatrix} \begin{bmatrix} P_t(t_{n+1}) \\ \vdots \\ P_t(t_{n+N_y}) \end{bmatrix} \\
- \lambda_{3i} \frac{2}{P_i^{max}} \sum_{j=1,j \neq i}^p \begin{bmatrix} a_{ij}(t_n) \frac{P_t(t_{n+1-\tau_{ij}})}{P_j^{max}} \end{bmatrix} \cdots \sum_{j=1,j \neq i}^p \begin{bmatrix} a_{ij}(t_n) \frac{P_t(t_{n+N_u-\tau_{ij}})}{P_j^{max}} \end{bmatrix} \right\} 
\]

Where \( \Gamma_i \) is defined according to (A4.4).

\[
\Gamma_i(t_n) = \sum_{j=1,j \neq i}^p a_{ij}(t_n) \tag{A4.4}
\]

From (A4.3) it is possible to state matrices \( H_i \) and \( F_i \) required by the cost function of the canonical QP problem defined by (3.17) and rewrote as (A4.5).

\[
\text{minimize} \quad J_i(t_n) := \frac{1}{2} \bar{X}^T_i H_i \bar{X}_i + F_i^T \bar{X}_i \\
\text{subject to} \quad A_i \bar{X}_i \leq B_i \\
A_{eq,i} \bar{X}_i = B_{eq,i} \tag{A4.5}
\]

Therefore:

\[
H_i(t_n) = \begin{bmatrix} 2\lambda_{1i} \Pi_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_u \times N_y} & 2\lambda_{2i} \Pi_{N_u} & 0_{N_u \times N_y} & 0_{N_u \times N_y} & 0_{N_u \times N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & 0_{N_y} \\
\end{bmatrix}_{((4N_y+N_u) \times (4N_y+N_u))}
\]

\[
F_i(t_n) = \begin{bmatrix} -2\lambda_{1i} \omega_0 \Pi_{1 \times N_y} & 0_{1 \times N_u} & 0_{1 \times N_y} & 0_{1 \times N_y} & 2\lambda_{3i} \frac{\Gamma_i(t_n)}{P_i^{max}} \Pi_{N_y} \end{bmatrix}_{(1 \times (4N_y+N_u))}^T
\]

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Where

\[
\mathbb{C}_{pi}(t_n) = \left[ - \sum_{j=1, j\neq i}^p \frac{a_{ij}(t_n)P_j(t_{n+1-\bar{t}_j})}{P_j \max} \ldots - \sum_{j=1, j\neq i}^p \frac{a_{ij}(t_n)P_j(t_{n+N_y-\bar{t}_j})}{P_j \max} \right]_{(1 \times N_y)}
\]

\[
\mathbb{I}_{1 \times N_y} = [1 \ldots 1]
\]

(A4.6)

- **Equality constraints:** Matrices \( A_{eq,i} \), and \( B_{eq,i} \) are built from equations (3.10), (3.11), and (3.12), and also, including operational constraints (3.13), and (3.14), which are rewrote as the set of equations (A4.7) to (A4.11):

\[
\omega_i(t_{n+k}) = \omega_i(t_{n+k-1}) + M_{\text{pow}}[P_i(t_{n+k}) - P_i(t_{n+k-1})] + \Delta \omega_{s,i}(t_{n+k-1})
\]

(A4.7)

\[
\delta \theta_i(t_{n+k}) = \delta \theta_i(t_{n+k-1}) + T_{se} \left[ \omega_i(t_{n+k}) - \omega_i^*(t_n) \right]
\]

(A4.8)

\[
P_i(t_{n+k}) = P_i(t_n) + [\delta \theta_i(t_{n+k}) - \delta \theta_i(t_n)] B_i V_i(t_n) V_i^* (t_n) \cos(\delta \theta_i(t_n))
\]

(A4.9)

\[
\bar{\omega}_i(t_{n+k}) = \frac{\omega_i(t_{n+k}) + \sum_{j=1}^p a_{ij}(t_n) \omega_j(t_{n+k-\bar{t}_j})}{1 + \sum_{j=1}^p a_{ij}(t_n)}
\]

(A4.10)

\[
\bar{\omega}_i(t_{n+N_y}) = \omega_0
\]

(A4.11)

Manipulating the predictive droop equation (A4.7), moving the predicted terms to the left side, and evaluating along the prediction horizon, is stated:

\[
\omega_i(t_{n+k}) - M_{\text{pow}}[P_i(t_{n+k}) - P_i(t_{n+k-1})] - \Delta \omega_{s,i}(t_{n+k-1}) = \omega_i(t_n) - M_{\text{pow}}[P_i(t_n)] \quad k = 1
\]

\[
[\omega_i(t_{n+k}) - \omega_i(t_{n+k-1})] - M_{\text{pow}}[P_i(t_{n+k}) - P_i(t_{n+k-1})] - \Delta \omega_{s,i}(t_{n+k-1}) = 0 \quad \forall 1 < k \leq N_u
\]

\[
[\omega_i(t_{n+k}) - \omega_i(t_{n+k-1})] - M_{\text{pow}}[P_i(t_{n+k}) - P_i(t_{n+k-1})] = 0 \quad \forall N_u < k \leq N_y
\]

Rewriting in matrix notation, (A4.12) is stated:

\[
\begin{bmatrix}
-\mathbb{I}_{N_u} & 0_{N_y \times N_u} \\
\end{bmatrix}
\begin{bmatrix}
\Delta \omega_{s,i}(t_n) \\
\Delta \omega_{s,i}(t_{n+N_u-1}) \\
\end{bmatrix}
+ \begin{bmatrix}
\mathbb{T}_{N_y} \\
\end{bmatrix}
\begin{bmatrix}
\omega_i(t_{n+1}) \\
\vdots \\
\omega_i(t_{n+N_y}) \\
\end{bmatrix}
\]

(A4.12)

\[
+ \begin{bmatrix}
- M_{\text{pow}} \mathbb{T}_{N_y} \\
\end{bmatrix}
\begin{bmatrix}
P_i(t_{n+1}) \\
\vdots \\
P_i(t_{n+N_y}) \\
\end{bmatrix}
= \begin{bmatrix}
\omega_i(t_n) - M_{\text{pow}}[P_i(t_n)] \\
0 \\
\vdots \\
0 \\
\end{bmatrix}
\]

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Where $T$ is a Toeplitz matrix defined by (A4.13), and is used to manage the difference between a predicted value and its previous one; and $(\mathbb{I}0)$, defined by (A4.14), represents an identity matrix and a zero matrix vertically concatenated.

\[
T_{Ny} = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
-1 & 1 & \cdots & 0 \\
0 & -1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}_{(Ny \times Ny)} \tag{A4.13}
\]

\[
(\mathbb{I}_{Nu} | 0)_{Ny \times Nu} = \begin{bmatrix}
\mathbb{I}_{Nu} \\
0_{(Ny-Nu) \times Nu}
\end{bmatrix}_{(Ny \times Nu)} \tag{A4.14}
\]

Applying the same procedure to angle phase equation (A4.8):

\[
\delta \theta_i(t_{n+k}) - T_{sec} \omega_i(t_{n+k}) = \delta \theta_i(t_n) - T_{sec} \omega_i^*(t_n) \quad k = 1
\]

\[
[\delta \theta_i(t_{n+k}) - \delta \theta_i(t_{n+k-1})] - T_{sec} \omega_i(t_{n+k}) = -T_{sec} \omega_i^*(t_n) \quad \forall 1 < k \leq Ny
\]

Therefore, in matrix notation:

\[
\begin{bmatrix}
\omega_i(t_{n+1}) \\
\vdots \\
\omega_i(t_{n+Ny})
\end{bmatrix}
- T_{sec} \mathbb{I}_{Ny}
\begin{bmatrix}
\delta \theta_i(t_{n+1}) \\
\vdots \\
\delta \theta_i(t_{n+Ny})
\end{bmatrix}
= \begin{bmatrix}
\delta \theta_i(t_n) - T_{sec} \omega_i^*(t_n) \\
\vdots \\
\delta \theta_i(t_n) - T_{sec} \omega_i^*(t_n)
\end{bmatrix} \tag{A4.15}
\]

Manipulating the power transfer equation (A4.9):

\[
P_i(t_{n+k}) - K_{\delta \theta}(t_n) \delta \theta_i(t_{n+k}) = P_i(t_n) - K_{\delta \theta}(t_n) \delta \theta_i(t_n) \quad \forall 1 \leq k \leq Ny
\]

Using matrix notation:

\[
-K_{\delta \theta}(t_n) \mathbb{I}_{Ny}
\begin{bmatrix}
\delta \theta_i(t_{n+1}) \\
\vdots \\
\delta \theta_i(t_{n+Ny})
\end{bmatrix}
+ \mathbb{I}_{Ny}
\begin{bmatrix}
P_i(t_{n+1}) \\
\vdots \\
P_i(t_{n+Ny})
\end{bmatrix}
= \begin{bmatrix}
P_i(t_n) - K_{\delta \theta}(t_n) \delta \theta_i(t_n) \\
\vdots \\
P_i(t_n) - K_{\delta \theta}(t_n) \delta \theta_i(t_n)
\end{bmatrix} \tag{A4.16}
\]

Where $K_{\delta \theta}$ is defined by (A4.17), and updated at the beginning of each sampling period.

\[
K_{\delta \theta}(t_n) = B_i V_i(t_n) V_i^*(t_n) \cos(\delta \theta_i(t_n)) \tag{A4.17}
\]

Applying the same procedure for operational constraints (A4.10) and (A4.11):

\[
\overline{\omega}_i(t_{n+k}) - \frac{\omega_i(t_{n+k})}{1 + \Gamma_i(t_n)} = \sum_{j=1}^{p} a_{ij}(t_n) \omega_j(t_{n+1-t_{ij}}) \frac{1}{1 + \Gamma_j(t_n)} \quad \forall 1 \leq k < Ny
\]

\[
\overline{\omega}_i(t_{n+1}) = \omega_0 \quad k = Ny \tag{A4.18}
\]
Using the matrix notation:

\[
\begin{bmatrix}
\omega_i(t_{n+1}) \\
\vdots \\
\omega_i(t_{n+N_y})
\end{bmatrix} + \begin{bmatrix}
-\frac{1}{1+\Gamma_i(t_n)}(I_{N_y}-1|0)_{N_y\times N_y} \\
\vdots \\
\omega_i(t_{n+N_y})
\end{bmatrix} = \begin{bmatrix}
\sum_{j=1}^{p} a_{ij}(t_n)\omega_j(t_{n+1-\hat{\tau}_{ij}}) \\
\vdots \\
\sum_{j=1}^{p} a_{ij}(t_n)\omega_j(t_{n+N_y-\hat{\tau}_{ij}}) \\
\end{bmatrix}
\]

(A4.19)

Where \(\Gamma_i(t_n)\) and \((I_{N_y}-1|0)_{N_y\times N_y}\) are defined according to (A4.4) and (A4.20), respectively.

\[
(I_{N_y}-1|0)_{N_y\times N_y} = \begin{bmatrix}
I_{N_y} & 0_{N_y-1\times 1} \\
0_{1\times N_y} & 0_{N_y}
\end{bmatrix}_{(N_y\times N_y)}
\]

(A4.20)

Therefore, matrices \(A_{eq,i}\) and \(B_{eq,i}\) can be built from equations (A4.12), (A4.15), (A4.16), and (A4.19) as follows.

\[
A_{eq,i}(t_n) = \begin{bmatrix}
I_{N_y} & 0_{N_y\times N_u} & -1_{1+N_y(t_n)}(I_{N_y}-1|0)_{N_y\times N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_y} & -(I_{N_u}|0)_{N_y\times N_u} & T_{N_y} & 0_{N_y} & -M_{p\omega,i}T_{N_y} \\
0_{N_y} & 0_{N_y\times N_u} & -T_{sec}I_{N_y} & 0_{N_y} & 0_{N_y} \\
0_{N_y} & 0_{N_y\times N_u} & 0_{N_y} & -K\delta\theta(t_n)I_{N_y} & I_{N_y}
\end{bmatrix}_{(N_y\times(4N_y+N_u))}
\]

(A4.21)

\[
B_{eq,i}(t_n) = \begin{bmatrix}
\sum_{j=1}^{p} a_{ij}(t_n)\omega_j(t_{n+1-\hat{\tau}_{ij}}) \\
\vdots \\
\sum_{j=1}^{p} a_{ij}(t_n)\omega_j(t_{n+N_y-\hat{\tau}_{ij}}) \\
\omega_i(t_n) - M_{p\omega,i}P_i(t_n) \\
0 \\
\vdots \\
\delta\theta_i(t_n) - T_{sec}\omega_i^*(t_n) \\
-T_{sec}\omega_i^*(t_n) \\
\vdots \\
P_i(t_n) - K\delta\theta(t_n)\delta\theta_i(t_n) \\
P_i(t_n) - K\delta\theta(t_n)\delta\theta_i(t_n)
\end{bmatrix}
\]

(A4.22)

\[\square\]

- **Inequality constraints:** Matrices \(A_i\) and \(B_i\) are built from inequality (3.15), rewrote as (A4.23).
\[ 0 \leq P_i(t_{n+k}) \leq P_{i\text{ max}} \quad (A4.23) \]

Expanding the inequalities along the prediction horizon:

\[
\begin{align*}
-P_i(t_{n+k}) &\leq 0 \\
\forall 1 \leq k \leq N_y \\
P_i(t_{n+k}) &\leq P_{i\text{ max}} \\
\forall 1 \leq k \leq N_y
\end{align*}
\]

Therefore:

\[
A_i(t_n) = \begin{bmatrix}
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & -I_{N_y} \\
0_{N_y} & 0_{N_y \times N_u} & 0_{N_y} & 0_{N_y} & I_{N_y}
\end{bmatrix}_{(2N_y \times (4N_y + N_u))} \quad (A4.24)
\]

\[
B_i(t_n) = \begin{bmatrix}
0_{N_y \times 1} \\
P_{i\text{ max}}I_{N_y \times 1}
\end{bmatrix}_{(2N_y \times 1)} \quad (A4.25)
\]
A5 Triphase® Microgrids Emulator

The Triphase® Microgrids Emulator, which is placed on the microgrids control lab on the University of Chile, is a system composed of two AC/AC units, PMF15F120 and PMF5F60R, and one DC/AC unit, PMF5F42R. Each unit uses a real time target (RTT), based on Unix, as a control unit.

Unit PMF15F120 has two independent power modules configurable as output modules, which are used as DG$_1$ and DG$_2$ in the experimental setup presented in section 4.2. The DG$_3$ is deployed in unit PMF5F60, which has only one output power module. Each DG’s output can be configured as an LC, LCC, LCL or LCLC filter, depending on whether it operates in voltage source or current source mode, respectively.

There are 4 communication networks for each DG. A real time EtherCat network is established among the RTT, the power electronics inverters, and the unit instrumentation (analog and digital signals), using UTP wires as physical connection. A conventional wired TCP/IP network is used to connect the engineering station and the RTT, being possible to manage few RTTs (assigned to different units), from the same engineering station, using a Matlab-Simulink® interface. The basic architecture of Triphase units is shown in Fig. A5.1, whereas the parameters of the power modules used, are shown in Table A5.2.

A third communication network is between the RTT and external measurement modules. Two kits of three voltage sensors are mounted downstream of the coupling inductances $L_1$ and $L_2$, and a optical fiber loop is established among these modules and the RTT of PMF15F120. One additional kit with three voltage sensors is used after $L_3$ to send the measurements to the RTT of unit PMF5F60; in this case the measurement kit and the RTT use a EtherCat network to establish communication. The specifications of used sensors are presented in [72].

Finally, the forth network allows the information sharing among DGs through an optical ring. Considering that this is a fast channel, several communication phenomena can be
Table A5.2: *Triphase®* Power Modules Specifications

<table>
<thead>
<tr>
<th>Unit</th>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMF15F120</td>
<td>Nominal Power</td>
<td>15KVA</td>
</tr>
<tr>
<td></td>
<td>Input Voltage</td>
<td>180–400 V_{RMS}[L-L]@50–60 Hz</td>
</tr>
<tr>
<td></td>
<td>Switching Frequency</td>
<td>8–16 KHz</td>
</tr>
<tr>
<td></td>
<td>Output Voltage</td>
<td>0–480 V_{RMS}[L-L]</td>
</tr>
<tr>
<td></td>
<td>Output Current</td>
<td>6 × 16 A_{RMS}</td>
</tr>
<tr>
<td></td>
<td>Maximum DC-Link Voltage</td>
<td>700 V_{DC}</td>
</tr>
<tr>
<td>PMF5F60</td>
<td>Nominal Power</td>
<td>5KVA</td>
</tr>
<tr>
<td></td>
<td>Input Voltage</td>
<td>220–400 V_{RMS}[L-L]@50–60 Hz</td>
</tr>
<tr>
<td></td>
<td>Switching Frequency</td>
<td>14–16 KHz</td>
</tr>
<tr>
<td></td>
<td>Output Voltage</td>
<td>0–480 V_{RMS}[L-L]</td>
</tr>
<tr>
<td></td>
<td>Output Current</td>
<td>3 × 8 A_{RMS}</td>
</tr>
<tr>
<td></td>
<td>Maximum DC-Link Voltage</td>
<td>700 V_{DC}</td>
</tr>
</tbody>
</table>

emulated and its impact over the microgrid performance can be evaluated. The network diagram used on the experimental setup is shown in Fig. A5.2.

Figure A5.2: Network Architecture for Experimental Setup.
Bibliography


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